CR-134042

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

APOLLO

GUIDANCE, NAVIGATION AND CONTROL

MIT CHARLES STARK DRAPER LABORATORY
CAMBRIDGE MASSACHUSETTS 02139

(NASA-CR-134042) CANDIDATE CONFIGURATION TRADE STUDY, STELLAR-INERTIAL MEASUREMENT SYSTEM (SIMS) FOR AN EARTH OBSERVATION SATELLITE (EOS), ADDENDUM (Massachusetts Inst. of Tech.) 96 ~~95~~ p HC $6.75 CSCL 22B N73-33805
Unclas
G3/31 18885

95

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

APOLLO

GUIDANCE, NAVIGATION AND CONTROL

Approved: P G Felleman Date: 30 Aug 1973
P.G. FELLEMAN

Approved: P G Felleman for Date: 30 Aug 1973
N.E. SEARS

Approved: David G. Hoag Date: 30 Aug 73
D.G. HOAG

ADDENDUM TO

R-741

FINAL REPORT
CANDIDATE CONFIGURATION TRADE STUDY,
STELLER-INERTIAL MEASUREMENT SYSTEM (SIMS)
FOR AN EARTH OBSERVATION SATELLITE (EOS)

by

Robert White and Fred Grant

30 June 1973

MIT CHARLES STARK DRAPER LABORATORY

CAMBRIDGE, MASSACHUSETTS, 02139

ACKNOWLEDGEMENT

This report was prepared under DSR Project 55-39200, sponsored by the Lyndon B. Johnson Space Center of the National Aeronautics and Space Administration through Task Order No. 42 of Contract NAS9-4065.

Technical direction of the effort was provided by Mr. John W. Kelly and Dr. Arun K. Guha of NASA Goddard Space Flight Center.

Technical assistance and supporting authorship in this report was provided by Milton B. Adams and Eliezer Gai Geisler.

The report was typed by Phyllis Amsler and Debbie Briggs. Illustrations were prepared by William Eng and David Farrar.

The publication of this report does not constitute approval by the National Aeronautics and Space Administration of the findings or the conclusions contained herein. It is published only for the exchange and stimulation of ideas.

ADDENDUM TO

R-741

FINAL REPORT
CANDIDATE CONFIGURATION TRADE STUDY, STELLAR-INERTIAL MEASUREMENT SYSTEM (SIMS) FOR AN EARTH OBSERVATION SATELLITE (EOS)

ABSTRACT

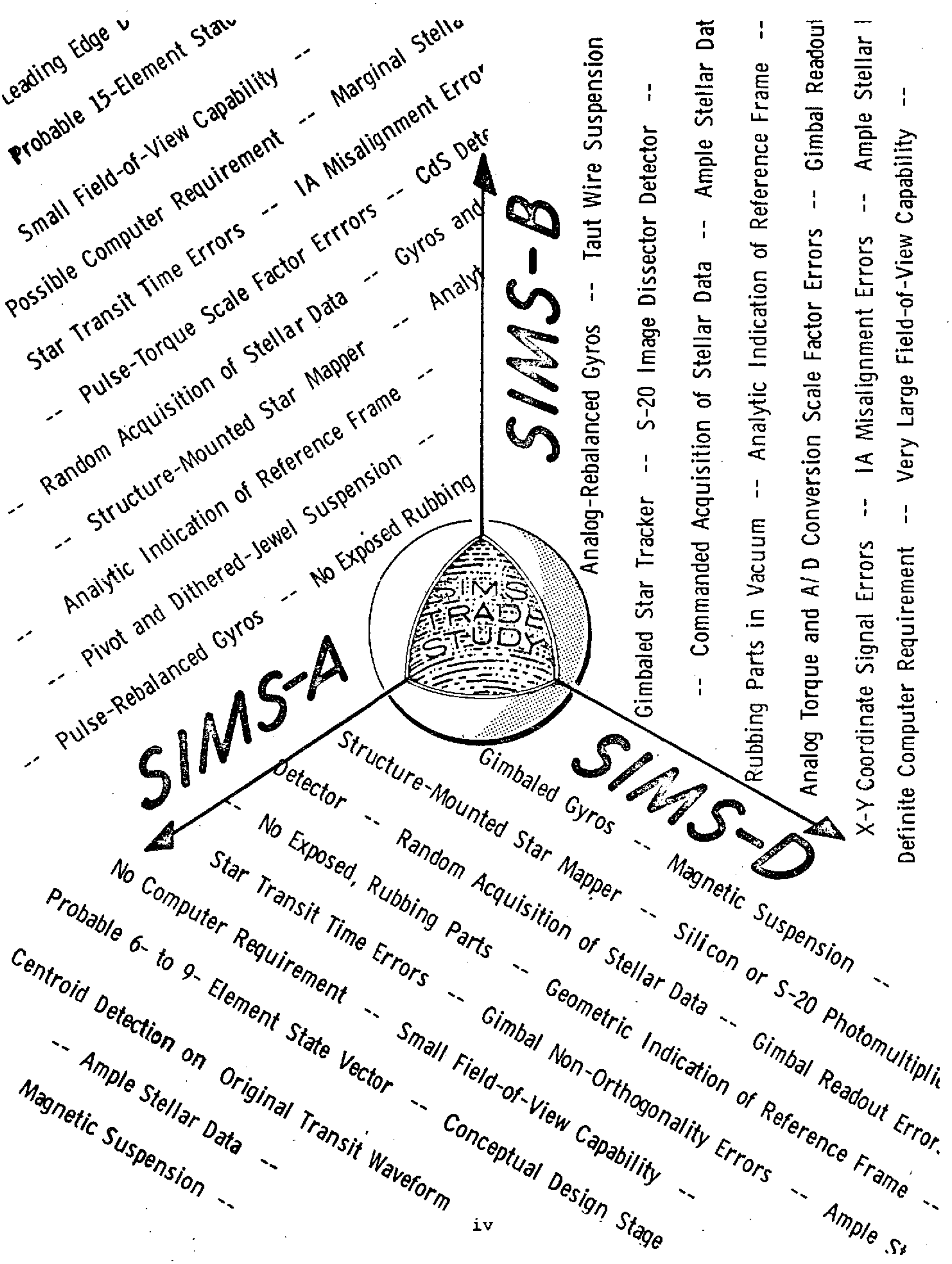
A two year trade study was performed by the Charles Stark Draper Laboratory Division of the Massachusetts Institute of Technology for the NASA L. B. Johnson Space Center, under the technical direction of the NASA Goddard Space Flight Center.

Ten candidate SIMS configurations were originally defined in the first interim technical report in November 1971 and reduced to three in the second such report in February 1972. The latter three were then studied in depth in the third interim technical report in June 1972. Afterwards, two additional studies were performed where one involved a more detailed error analysis of the SIMS-A candidate and the other was an investigation of the use of known landmarks in the payload sensor imagery to estimate spacecraft attitude in the event of star sensor failure for the SIMS-A and SIMS-B candidates. The results of these two studies were reported on in the Final Report in January 1973.

This addendum to the Final Report gives the results of the latest study which was an investigation of the effects of spacecraft orbital ephemeris and attitude errors on the ability to determine the locations of unknown landmarks with respect to known landmarks in the payload sensor imagery.

by Robert White
Fred Grant

30 June 1973


SIMS TRADE STUDY
SIMS-A
SIMS-B
SIMS-D
Leading Edge D
Probable 15-Element Stat
Small Field-of-View Capability -- Marginal Stella
Possible Computer Requirement -- IA Misalignment Erro
Star Transit Time Errors -- CdS Dete
-- Pulse-Torque Scale Factor Errrors -- Gyros and
Random Acquisition of Stellar Data -- Analyt
-- Structure-Mounted Star Mapper --
-- Analytic Indication of Reference Frame --
-- Pivot and Dithered-Jewel Suspension --
-- Pulse-Rebalanced Gyros -- No Exposed Rubbing
Analog-Rebalanced Gyros -- Taut Wire Suspension
Gimbaled Star Tracker -- S-20 Image Dissector Detector --
-- Commanded Acquisition of Stellar Data -- Ample Stellar Dat
Rubbing Parts in Vacuum -- Analytic Indication of Reference Frame --
Analog Torque and A/D Conversion Scale Factor Errors -- Gimbal Readout
X-Y Coordinate Signal Errors -- IA Misalignment Errors -- Ample Stellar
Definite Computer Requirement -- Very Large Field-of-View Capability --
Gimbaled Gyros -- Magnetic Suspension --
Structure-Mounted Star Mapper -- Silicon or S-20 Photomultipli
Detector -- Random Acquisition of Stellar Data -- Gimbal Readout Error.
-- No Exposed, Rubbing Parts -- Geometric Indication of Reference Frame --
Star Transit Time Errors -- Gimbal Non-Orthogonality Errors -- Ample St
No Computer Requirement -- Small Field-of-View Capability --
Probable 6- to 9- Element State Vector -- Conceptual Design Stage
Centroid Detection on Original Transit Waveform
-- Ample Stellar Data --
Magnetic Suspension --

PREFACE

This addendum to the Final Report presents an investigation of the effects of spacecraft orbital ephemeris and attitude errors on the ability to determine the locations of unknown landmarks with respect to known landmarks in the payload sensor imagery of an Earth Observation Satellite in a sun-synchronous orbit at 540 nautical miles altitude. Most of the effort was concerned with the ability to determine landmark locations in an observational pass over the continental USA.

The data in this report gives the sensitivity of landmark location error to errors in spacecraft ephemeris, attitude, gyro bias drift, and higher harmonics in the Earth's gravitational field. Also presented is data showing the relative roles that the several error sources have on landmark location determination. Finally, some results are given showing the effects of typical sets of errors in the smoothed estimates of attitude and gyro bias drift for SIMS-A. It has been found that the errors in the smoothed estimates of attitude and gyro bias drift do not affect landmark location determination as much as some people might suspect.

CONTENTS

CONTENTS (Cont.)

CONTENTS (Cont.)

ILLUSTRATIONS

TABLES

TABLES (Cont.)

TABLES (Cont.)

TABLES (Cont.)

SECTION 1

INTRODUCTION AND SUMMARY

1.1 INTRODUCTION

This report has been prepared as an addendum to the previously published Final Report[177*] and covers the studies performed between 1 April 1973 and 30 June 1973 by the Charles Stark Draper Laboratory Division of the Massachusetts Institute of Technology (MIT/CSDL) on the "Candidate Configuration Trade Study--Stellar-Inertial Measurement System (SIMS) for a Proposed Earth Observation Satellite (EOS)" for the NASA Goddard Space Flight Center (GSFC).

Three prior Interim Technical Reports[85, 141, 169] and a Final Report[177] have been published. Excerpts from MIT/CSDL Technical Proposal No. 71-173, dated June 1971, including the statement of work for the first eleven months of this effort, were provided as Appendix A of Reference 85. Excerpts from MIT/CSDL Technical Proposal No. 72-176, dated 16 May 1972, including the statement of work for the following ten months of the effort, were provided as Appendix E of Reference 169. The final three months of this effort were spent on additional studies proposed in MIT/CSDL Technical Proposal No. 73-132 and reported on in this addendum to the Final Report. Two new studies were to be performed in this final three month effort which are as follows:

1) Landmark Location Study - Investigate the effects of spacecraft orbital error, in addition to attitude determination errors, on the ability to determine the locations of unknown landmarks with respect to known landmarks in the payload sensor imagery.

* Superscripts refer to similarly numbered references in the reference section (Section 6) of this report or the reference section of the previous reports of the SIMS Trade Study.

2) Active and Passive Landmark Mechanization Study - Perform a preliminary survey of the various techniques or devices on ground which might be suitable for obtaining measurement data for EOS orbit and attitude determination.

The data presented in this addendum to the Final Report represents the results of the Landmark Location Study. Due to other commitments of key personnel, MIT/CSDL was unable to perform the Active and Passive Landmark Mechanization Study during this three month period. Separate negotiations are now underway to perform this study at a later time.

1.2 BACKGROUND OF SIMS TRADE STUDY

Four categories of SIMS candidate configurations were originally required to be evaluated and compared in the SIMS Trade Study. These were designated as A, B, C, and D. Later, E was added and D was subdivided into sub-groups. Thus at the time of the First Interim Technical Report, 10 candidates were defined as potential SIMS design approaches. These were eventually narrowed down to 3 candidates in the Third Interim Technical Report and were simply designated as SIMS-A, SIMS-B, and SIMS-D. The accuracies given for the three candidates in the Third Interim Technical Report were obtained by performing a covariance analysis with an optimal smoother. This data provided a statistical indication of the accuracy possible after smoothing star and gyro rate measurements over a number of satellite orbits. In the error studies presented in the Final Report a more detailed error study was performed on the SIMS-A candidate. Smoothed results were obtained for the estimates of spacecraft attitude and gyro bias drift and also for the uncertainties in these estimates. The Final Report also presented the results of an investigation into the use of known landmarks in the payload sensory imagery to estimate attitude in the event of star sensor failure in SIMS-A and SIMS-B.

1.3 SUMMARY

This subsection briefly summarizes the principal data given in this report. This report was primarily concerned with determining how errors in the estimation of spacecraft attitude and orbital ephemeris would affect the ability to determine the locations of landmarks in the payload sensor imagery during an observational pass over the continental USA. The effect of incomplete modeling of the Earth's non-spherical gravitational field and of the uncertainties in the harmonic coefficients of that field are also studied. The satellite orbit was assumed to be a sun-synchronous orbit with an altitude of 540 nautical miles and an inclination of 99 degrees. The scan beam of the payload sensor generated a swath of imagery 100 nautical miles wide by sweeping back and forth across the satellite ground track to a maximum angle of 5.3° to each side of the orbital plane.

Section 2 of this report presents the analytical and supporting equations of this study. Section 3 gives the error study results which are roughly divided as follows:

1) Sensitivity Studies - Data is given showing the sensitivity of landmark error to error in each component of satellite attitude, orbital ephemeris, and gyro bias drift. Included also are the effects of attitude libration and various gyro errors and noises.

2) Gravitational Modeling - Data is given showing the landmark errors caused by the higher harmonics in the Earth's gravitational field. Included also are the effects of uncertainties in these harmonics.

3) Effect of Smoothed Estimates of Attitude and Gyro Bias Drift - Data is given showing the landmark errors caused by errors

in the smoothed estimates of attitude and gyro bias drift of previous SIMS studies.

4) Effect of Combined Errors in Spacecraft Attitude and Orbital Ephemeris - Data is given showing the combined effect of nominal errors in satellite attitude and orbital ephemeris.

Section 4 presents some of the conclusions drawn from this study and Section 5 gives some recommended future studies. In Appendix A some corrections are given for the previously published Final Report[177].

SECTION 2

LANDMARK LOCATION STUDY ANALYSIS

2.1 INTRODUCTION

The primary objective of the landmark location study was to determine the effects of spacecraft orbital ephemeris and attitude errors on the ability to determine the locations of unknown landmarks with respect to known landmarks in the payload sensor imagery. Sufficient data was generated to show the sensitivity of landmark location determination to errors in each component of the spacecraft orbital ephemeris and attitude. This data can be of great value in indicating what accuracies are required in the orbital ephemeris and spacecraft attitude in order to achieve a given accuracy in landmark location determination. No attempt was made to correct (or update) our knowledge of the orbital ephemeris and spacecraft attitude using the observations made on known landmarks since this is a different problem and should be given separate treatment in future studies using optimal filter and/or smoother techniques (see Section 5).

In the present study the observed discrepancy in the location of a known landmark is used to correct our knowledge of the locations of other arbitrary but unknown features seen at different times in the payload sensor imagery. This gives us an indication of how well we can determine the location of an unknown landmark (or feature) with respect to a known landmark. During the period between observations of the known and unknown landmarks, the error in each component of the satellite orbital ephemeris and attitude will change in a prescribed manner; and it should be noted that it is the 'change' in the errors of spacecraft orbital position and attitude, and not the errors in these quantities, which will have the most significant effect on the relative landmark location accuracy.

One other objective of the landmark location study was to determine the effects of uncertainties in the Earth's gravitational model on the propagation of the orbital ephemeris for an Earth Observation Satellite (EOS). Coupled with this was a desire to determine how complete the gravitational model should be in the landmark location problem. Today, the Earth's gravitational model is fairly well known and can be represented by a large number of terms in a series expansion. Most of these terms are relatively small and are not required in this particular application.

In this study most of the emphasis was placed on landmark location accuracy within the continental USA (between 30° and 50° latitude). Figure 2-1 shows a typical observational pass over the USA. The EOS satellite is assumed to be in a circular orbit with an altitude of 540 nautical miles and an inclination of 99 degrees. Onboard the satellite there is assumed to be a thematic mapper (or multi-spectral scanner) whose beam is directed downwards and sweeps back and forth across the ground track to generate a swath of imagery 100 nautical miles wide. Under nominal attitude conditions, the spacecraft body axes (X_B, Y_B, Z_B) are as shown in Figure 2-1, with X_B in the direction of satellite motion and Z_B pointing to nadir. The scanner beam sweeps back and forth about X_B with a maximum excursion of 5.3° with respect to Z_B. The angle between the beam and Z_B is denoted as ϵ and is positive when in the same direction as Y_B.

Most of the performance results of this study are for an observational pass which starts at 50° latitude with specified errors in satellite position, velocity, attitude, and gyro bias drift. The satellite position and velocity at any later point are obtained by solving the equations of motion. The spacecraft attitude at any later point is obtained by using a fourth order Runge Kutta integration of the indicated body rates of three body mounted gyros. The input axes of the gyros are assumed to be oriented in the same way as in SIMS-A and SIMS-B, which is parallel to the spacecraft body axes X_B, Y_B and Z_B. A realistic simulation was used for the gyros, including the effects of bias drift error, random drift, scale factor error, input axis misalignment, and quantization.

It is important to note that no updates were made in the satellite orbital ephemeris and attitude during the relatively short observational pass

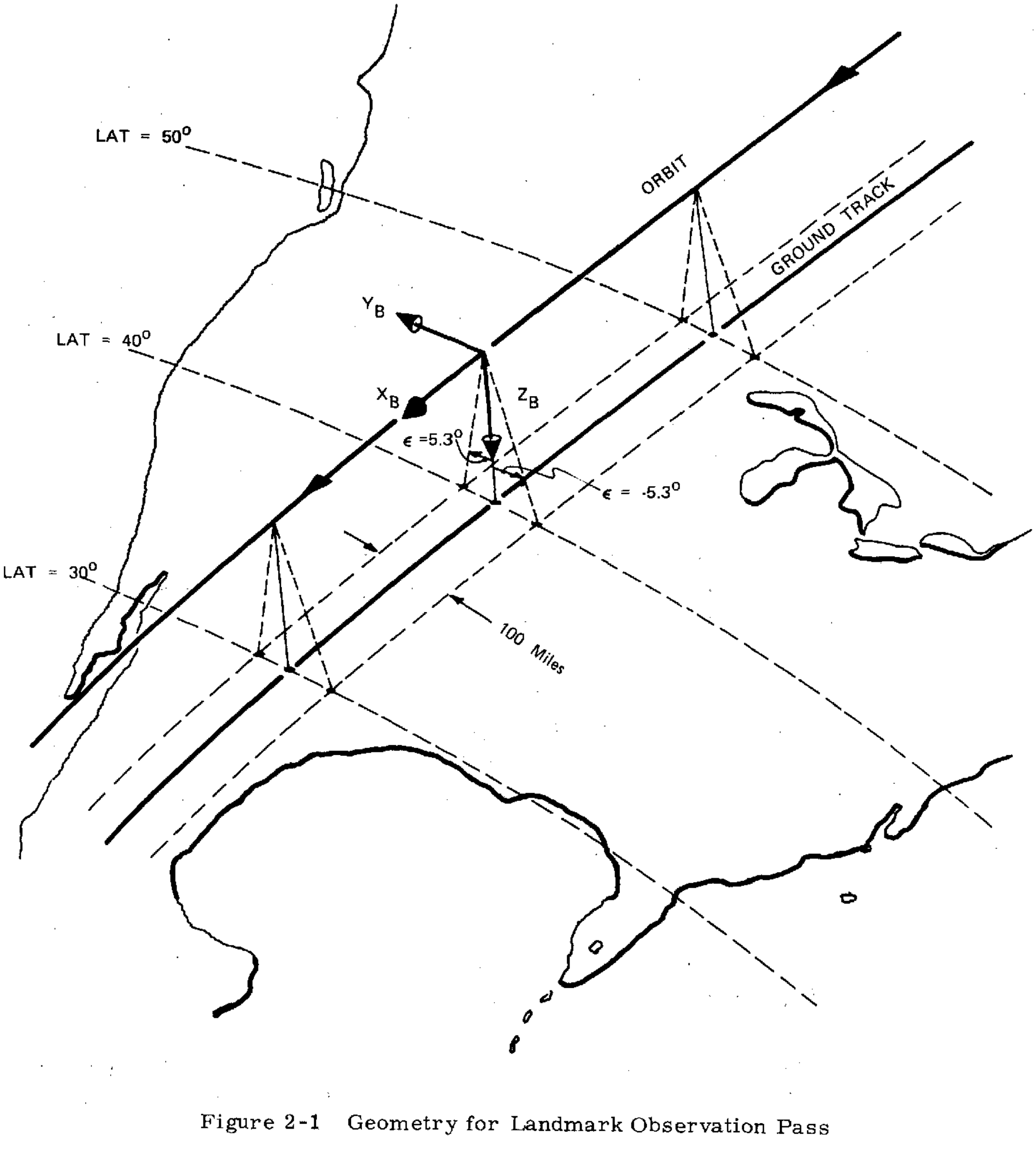


Figure 2-1 Geometry for Landmark Observation Pass

over the USA since it was felt that such updates are usually random in character and would probably introduce more error in the relative landmark location determination than the changes in the errors of the assumed orbital ephemeris and attitude. A possible exception to the above might be the case where use is made of a smoothed well-behaved satellite position and attitude history based on batch processing.

It should also be noted that no consideration was given to the effect of thematic mapper errors on landmark location determination since this was beyond the scope of this effort.

2.2 RELATIONSHIP BETWEEN LANDMARK LOCATION AND SATELLITE ATTITUDE AND POSITION

It is desired to obtain the equations relating spacecraft attitude and position to landmark location. We first obtain equations for the landmark position vector as a function of the orbital position vector and the line of sight vector of the scan beam at the time of observation of the landmark.

Figure 2-2 shows the beam vector, $\underline{s}$, the landmark position vector, $\underline{l}$, and the satellite position vector, $\underline{r}$. Also shown are the satellite altitude, H, and the instantaneous deviation angle, δ, of the scan beam vector from the satellite position vector (or local vertical). From the figure we have:

$$\underline{l} = \underline{r} + \underline{s} \qquad (2\text{-}1)$$

where all vectors are assumed to be in the basic inertial coordinate system, which is Earth centered with its Z-axis along the north pole and the other two axes are in the equatorial plane.

Since the deviation angle, δ, will be small in the present application, a flat Earth can be assumed and Equation 2-1 can be expressed as follows:

$$\underline{l} = \underline{r} + (H/\cos\delta)\,\underline{u}_{LOS} \qquad (2\text{-}2)$$

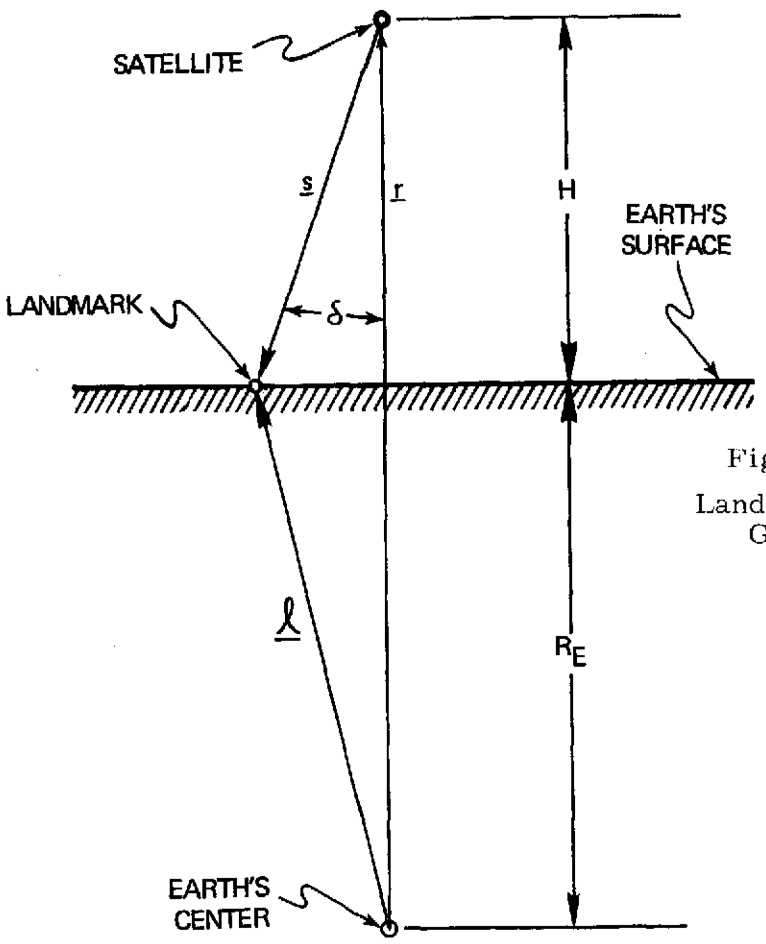


Figure 2-2

Landmark Vector Geometry

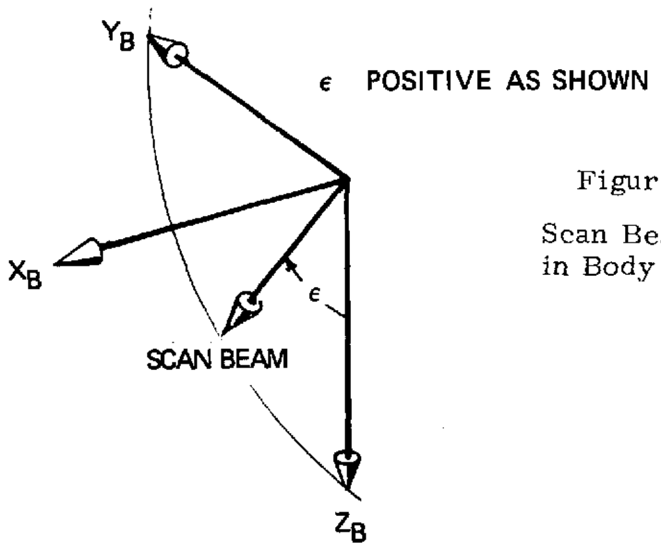


Figure 2-3

Scan Beam Direction in Body Coordinates

where

$$\underline{u}_{LOS} = \text{UNIT}(\underline{s})$$

$$\cos\delta = \underline{u}_{LOS} \cdot \left[-\text{UNIT}(\underline{r}) \right]$$

The direction of the scan beam in spacecraft body coordinates at any given instant is completely defined by the scan beam angle, ϵ , as shown in Figure 2-3. In basic inertial coordinates, the unit vector defining the direction of the beam is given by:

$$\underline{u}_{LOS} = \underline{z}_B \cos\epsilon + \underline{y}_B \sin\epsilon \tag{2-3}$$

where $\underline{y}_B$ and $\underline{z}_B$ are unit vectors defining the directions of the Y and Z axes of the spacecraft body in basic inertial coordinates and are obtained as follows:

$$\underline{y}_B = T_{IO} \; T_{OB} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \qquad \underline{z}_B = T_{IO} \; T_{OB} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{2-4}$$

where T_{OB} is the transformation matrix from body to orbital coordinates and T_{IO} is the transformation matrix from orbital to basic inertial coordinates. The relationships between the Body, Orbital and Basic Inertial Coordinate Systems are shown in Figures 2-4 and 2-5. The equations for T_{IO} and T_{OB} are:

$$T_{IO} = \begin{bmatrix} c\Omega & -s\Omega\, ci & s\Omega\, si \\ s\Omega & c\Omega\, ci & -c\Omega\, si \\ O & si & ci \end{bmatrix} \tag{2-5}$$

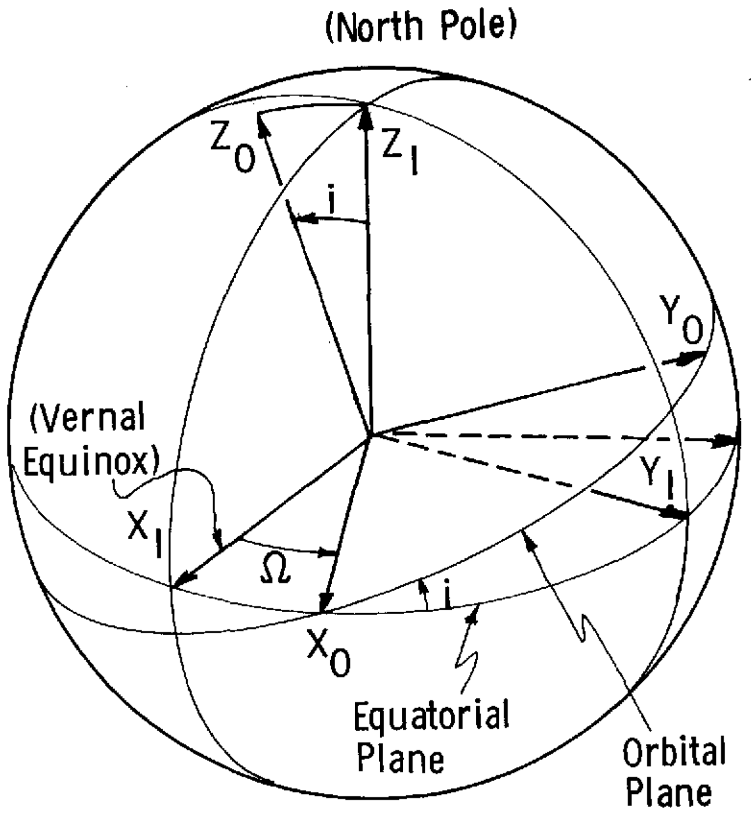


Figure 2-4 Basic Inertial and Orbital Coordinate Systems

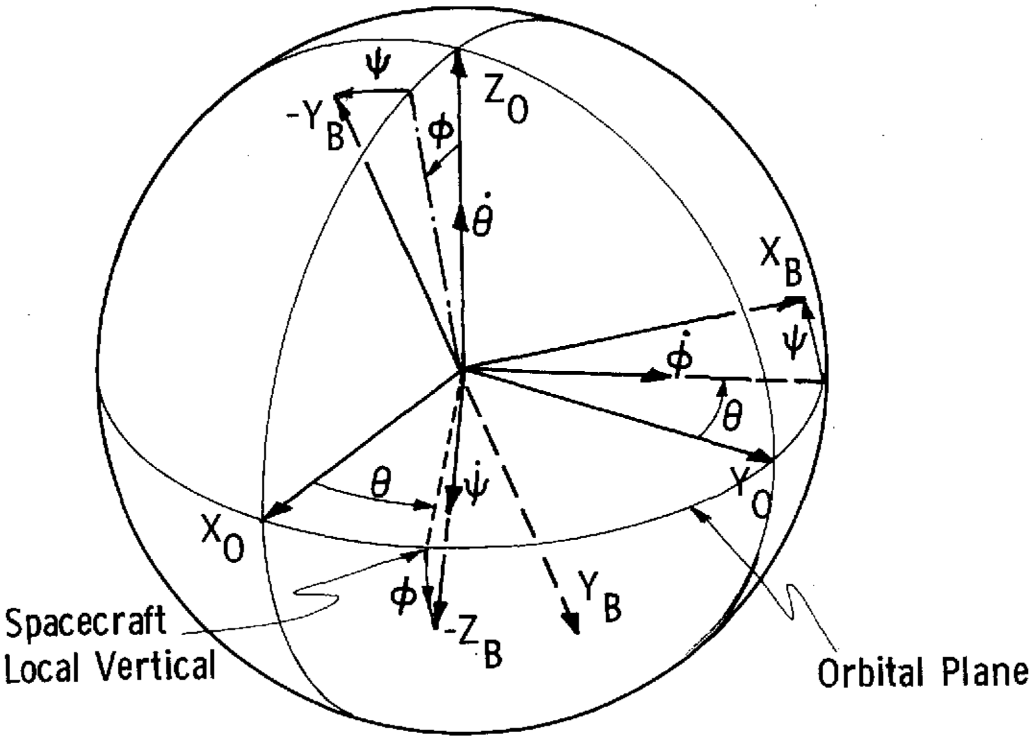


Figure 2-5 Orbital and Spacecraft Body-Fixed Coordinate Systems

$$T_{OB} = \begin{bmatrix} -c\psi\ s\theta & -s\psi\ s\theta & -c\phi\ c\theta \\ +s\psi\ s\phi\ c\theta & -c\psi\ s\phi\ c\theta & \\ & & \\ c\psi\ c\theta & s\psi\ c\theta & -c\phi\ s\theta \\ +s\psi\ s\phi\ s\theta & -c\psi\ s\phi\ s\theta & \\ & & \\ s\psi\ c\phi & -c\psi\ c\phi & s\phi \end{bmatrix} \tag{2-6}$$

where s and c denote sine and cosine; Ω is orbital right ascension; i is orbital inclination; and θ, ϕ, and ψ are the spacecraft pitch, roll and yaw angles, respectively. Substitution of Equation 2-6 into Equation 2-4 gives:

$$\underline{y}_B = T_{IO} \begin{bmatrix} -s\psi\ s\theta - c\psi\ s\phi\ c\theta \\ s\psi\ c\theta - c\psi\ s\phi\ s\theta \\ -c\psi\ c\phi \end{bmatrix} \tag{2-7}$$

$$\underline{z}_B = T_{IO} \begin{bmatrix} -c\phi\ c\theta \\ -c\phi\ s\theta \\ s\phi \end{bmatrix} \tag{2-8}$$

Note that Equations 2-7 and 2-8 may be used to compute either the true or estimated body axis vectors depending on whether the true or estimated spacecraft attitude angles are used.

When the expression for $\underline{u}_{LOS}$ in Equation 2-3 is substituted into Equation 2-2, the following is obtained for the landmark position vector:

$$\underline{1} = \underline{r} + (H/\cos\delta) \left[\underline{z}_B \cos\epsilon + \underline{y}_B \sin\epsilon \right] \tag{2-9}$$

where $\underline{1}$ can either be the true or estimated landmark position vector depending on whether true or estimated values are used for $\underline{r}$, $\underline{z}_B$, and $\underline{y}_B$. In the present effort, both values of $\underline{1}$ are computed for each landmark. The difference between the two values represents the error in estimating the landmark location due to errors in the estimates of orbital position, $\underline{r}$, and spacecraft attitude, $\underline{z}_B$ and $\underline{y}_B$. In this study, most of the attention has been directed towards the horizontal components of the landmark error. These components may be computed relative to north-south and east-west axes from which the errors in latitude and longitude may be computed. However, the performance results in this report have been presented in cross-track (normal to the orbital plane) and down-range coordinates. In the tabular results these two terms have been shortened to 'track' and 'range', respectively. The track and range components of the landmark error were chosen in order to clarify certain interesting effects which are discussed later in the section on results (Section 3). The track and range components of the estimated or true landmark position vector may be computed in true local vertical coordinates as follows:

$$l_{TRK} = R_E \sin^{-1}\left\{\left[\text{UNIT } (\underline{l})\right] \cdot \left[\text{UNIT } (\underline{v} \times \underline{r})\right]\right\} \tag{2-10}$$

$$l_{RNG} = R_E \sin^{-1}\left\{\left[\text{UNIT } (\underline{l})\right] \cdot \left[\text{UNIT } (\underline{v})\right]\right\} \tag{2-11}$$

where $\underline{r}$ and $\underline{v}$ are the true satellite position and velocity vectors, respectively, and $\underline{l}$ is the estimated or true landmark position vector. In this case the track and range components of the error in the estimate of landmark location are:

$$e_{TRK} = l_{TRK_E} - l_{TRK_T} \tag{2-12}$$

$$e_{RNG} = l_{RNG_E} - l_{TRK_T} \tag{2-13}$$

where the subscripts E and T denote the estimated and true values, respectively.

In Section 2.6 a discussion is given of the technique used to correct the position estimates of arbitrary features (or unknown landmarks) observed during the observational pass. This correction is based on the values of e_{TRK} and e_{RNG} obtained for a single control point observed during the pass. After correction, the errors in the position estimates of the arbitrary features are considered to be an indication of how well these features can be mapped with respect to the control point.

The latitude and longitude of the landmark vector, $\underline{l}$, can be computed as follows:

$$\mathrm{LAT} = \tan^{-1}\left\{ l_Z \Big/ \sqrt{l_X^2 + l_Y^2} \right\} \tag{2-14}$$

$$\mathrm{LONG} = \mathrm{SIGN}\,(l_Y)\ \cos^{-1}\left\{ l_X \Big/ \sqrt{l_X^2 + l_Y^2} \right\} \tag{2-15}$$

where the subscripts denote the components of $\underline{l}$ in Earth fixed coordinates.

2.3 PROPAGATION OF STATE ESTIMATES OF SPACECRAFT ATTITUDE AND GYRO BIAS DRIFT

In the Third Interim Technical Report[169] and in the Final Report[177], considerable data was presented on the uncertainties and errors in the smoothed estimates of spacecraft pitch, roll, and yaw, and bias drift rate of each gyro. These errors were for a particular specified time in the data processing interval, usually the mid-point of the data interval.

The ground-rule adopted in the present error study was that there be no update of spacecraft attitude and gyro bias drift during the observational pass over the USA. At the start of an observational pass a given set of values (including errors) was assumed for pitch, roll, yaw, and gyro bias drift. In some cases these values represented the smoothed results of previous SIMS studies. Thereafter, during the observational pass, the spacecraft attitude (θ, ϕ and ψ) was obtained by performing a fourth order Runge Kutta integration of the following rates:

$$\begin{bmatrix} \dot\theta \\ \dot\phi \\ \dot\psi \end{bmatrix} = \begin{bmatrix} s\psi/c\phi & -c\psi/c\phi & 0 \\ c\psi & s\psi & 0 \\ s\phi\, s\psi/c\phi & -s\phi\, c\psi/c\phi & -1 \end{bmatrix} (\underline{\omega}_M - \underline{b}) \tag{2-16}$$

where $\underline{\omega}_M$ is the vector for the measured gyro rates which includes the effects of true gyro bias drift, bias and random errors in gyro scale factor, errors in gyro input axis alignment, and errors due to random drift (see subsection 2.2.2 of the Final Report[177]). The vector $\underline{b}$ represents the estimate of bias drift for each gyro, which is used to compensate for the bias drift present in $\underline{\omega}_M$.

It is important to note that the spacecraft attitude was propagated in the same manner as was done in the detailed error analysis of SIMS-A except that no updates were made with star measurements.

The true spacecraft attitude history used to generate most of the results in the present effort was nominal (i.e., the roll and yaw angles (ϕ and ψ) were zero, and the pitch angle (θ) changed at a constant orbital rate). However, in some cases, use was made of the following attitude history:

$$\begin{aligned}
\theta_T &= \omega t + \theta_L \sin(\omega_L t + 120^\circ) + \theta_0 \\
\phi_T &= \phi_L \sin(\omega_L t) + \phi_0 \\
\psi_T &= \psi_L \sin(\omega_L t + 240^\circ) + \psi_0
\end{aligned} \tag{2-17}$$

where θ_T, ϕ_T, and ψ_T are the true pitch, roll, and yaw angles, respectively; ω is the constant orbital rate; t is the time since t = 0; θ_0, ϕ_0, and ψ_0 are the attitude angles at t = 0; θ_L, ϕ_L, and ψ_L are the libration amplitudes (0.6° each); and ω_L is the libration angular rate (0.478 degrees per second).

2.4 PROPAGATION OF ORBITAL EPHEMERIS

An important, if not most significant, source of error in the present landmark location problem is the error in the knowledge of satellite position and velocity. This error is primarily due to errors in tracking system instrumentation and uncertainties in gravitational modeling, atmospheric drag, and tracking station location.

Various values have been quoted for the uncertainties in orbital ephemeris depending upon the extent and quality of ground tracking and data processing. For the purposes of this study it was indicated by GSFC that a satellite position uncertainty of 50 meters should be considered as nominal with most of this being in the down-range component. Based upon previous CSDL experience with uncertainties in orbital position and velocity, the following uncertainties were selected as being nominal for the components of position and velocity: 50 meters in down-range; 20 meters in altitude and cross-track; 0.05 meters per second in altitude rate; and 0.02 meters per second in cross-track and down-range rate. Actually, the selection of the above values as nominal is not too important since sufficient sensitivity results were generated to show the effect of different values of error in each component of the orbital ephemeris.

In the present study, a given set of errors in orbital position and velocity were assumed at the start of an observational pass over the USA. These errors were then propagated along the orbit to the points of interest.

There are several methods by which the errors in position and velocity may be computed for a later point in orbit. The one employed in the present study used a fourth-order Runge Kutta integration of the following equations of motion to obtain the position, $\underline{r}$, and velocity, $\underline{v}$, at some desired point for the case of no initial errors and for the case with initial errors. The difference between the two solutions represents the error at that point.

$$\dot{\underline{v}} = -\mu \frac{\underline{r}}{r^3} \qquad (2\text{-}18)$$

$$\dot{\underline{r}} = \underline{v} \qquad (2\text{-}19)$$

where μ is the gravitational constant.

Actually, there are simpler ways to compute the position and velocity at some later point in orbit when only a central force field is assumed for the Earth. One of these is the following closed-form vector solution:

$$\underline{r} = \left\{1 - \frac{r}{p}\left[1 - \cos(\Delta\theta)\right]\right\}\underline{r}_0 + \frac{r\,r_0}{\sqrt{\mu p}}\sin(\Delta\theta)\,\underline{v}_0 \qquad (2\text{-}20)$$

$$\underline{v} = \left\{\frac{\underline{r}_0 \cdot \underline{v}_0}{p\,r_0}\left[1 - \cos(\Delta\theta)\right] - \frac{1}{r_0}\sqrt{\frac{\mu}{p}}\sin(\Delta\theta)\right\}\underline{r}_0 + \left\{1 - \frac{r_0}{p}\left[1 - \cos(\Delta\theta)\right]\right\}\underline{v}_0 \qquad (2\text{-}21)$$

where $\underline{r}_0$ and $\underline{v}_0$ are the initial position and velocity vectors; $\underline{r}$ and $\underline{v}$ are the position and velocity vectors after a true anomaly change of $\Delta\theta$; and p is the parameter of the orbit. Although this method was not employed in the present effort, it has been presented as a point of interest.

A third method of computing the errors at some later point is to use the following approximate error propagation equations which are satisfactory for a circular or near circular orbit. In addition to saving computational time, the following equations are useful in demonstrating certain relationships between the errors:

$$e_{ALT} \cong e_{ALT_0}(2 - \cos\omega t) + e_{RNG_0}\sin\omega t + \dot{e}_{ALT_0}(1/\omega)\sin\omega t + \dot{e}_{RNG_0}(2/\omega)(1 - \cos\omega t) \qquad (2\text{-}22)$$

$$e_{RNG} \cong e_{ALT_0}(2\sin\omega t - 3\omega t) + e_{RNG_0}(2\cos\omega t - 1) - \dot{e}_{ALT_0}(2/\omega)(1 - \cos\omega t) + \dot{e}_{RNG_0}(1/\omega)(4\sin\omega t - 3\omega t) \qquad (2\text{-}23)$$

$$e_{TRK} \cong e_{TRK_0}\cos\omega t + \dot{e}_{TRK_0}(1/\omega)\sin\omega t \qquad (2\text{-}24)$$

where the subscripts ALT, RNG, and TRK denote the altitude, down-range, and cross-track components of the error, respectively; the subscript 0 denotes the initial error at $t = 0$; and ω is the spacecraft orbital rate. Note that Equations 2-22 through 2-24 give only the position errors at a later point in orbit since the velocity errors at that point do not affect the landmark location determination at that point. Later, in the discussions of Section 3, it will be found that these equations are very useful in explaining the performance results.

2.5 CONSIDERATION OF EARTH'S GRAVITATIONAL MODEL

In Section 2.4 consideration was given to propagation of the orbital ephemeris errors in the Earth's central force field (i.e. only the gravitational constant μ was assumed to be present). Although most of the performance results of this study were for this case, some data was generated showing the effects of higher harmonics in the Earth's gravitational potential. The primary reason for considering a more exact model of the Earth's gravitational potential was to determine how complete the model should be for the particular problem of landmark location determination in the continental USA.

The following material presents some of the basic equations used for modeling the Earth's gravitational field. These equations are based upon some important work by S. Pines in Reference 178.

The gravitational potential, U, can be expressed as:

$$U = \frac{\mu}{r}\left\{1 + \sum_{n=2}^{\infty}\sum_{m=0}^{n}\left(\frac{a}{r}\right)^n P_{n,m}(u)\left[C_{n,m}\cos m\lambda + S_{n,m}\sin m\lambda\right]\right\} \tag{2-25}$$

where

$C_{n,m}$ and $S_{n,m}$ are constants

$(C_{n,0} = J_n , S_{n,0} = 0)$

$$P_{n,m}(u) = \frac{(1 - u^2)}{2^n \; n\,!} \left(\frac{d}{du}\right)^{n+m} (u^2 - 1)^n$$

$u = z/r = \cos \phi$ (Latitude = $90^o - \phi$)

$\lambda = \tan^{-1} (y/x)$ = Longitude

a = Equatorial radius

r = Radial distance

μ = Gravitational constant

The general equation for the accelerating force vector is

$$\underline{f} = \frac{dU}{d\underline{x}} \tag{2-26}$$

or

$$\underline{f} = \frac{\partial U}{\partial r}\frac{dr}{d\underline{x}} + \frac{\partial U}{\partial u}\frac{du}{d\underline{x}} + \frac{\partial U}{\partial \lambda}\frac{d\lambda}{d\underline{x}} \tag{2-27}$$

Because of the existence of a singularity at $\sin \phi = 0$ that develops with the use of these equations, it was found convenient to change to a four parameter system. For this we have

$$\underline{r} = r \begin{bmatrix} s \\ t \\ u \end{bmatrix} \tag{2-28}$$

where s, t, and u are the direction cosines of $\underline{r}$. The following general expression for the force vector can then be obtained:

$$\underline{f} = -\frac{\mu}{r^2}\underline{u}_r + \left[\alpha_r \underline{u}_r + \alpha_1 \underline{i} + \alpha_2 \underline{j} + \alpha_3 \underline{k}\right] \tag{2-29}$$

= conic acceleration + disturbing acceleration

After evaluating the terms of the disturbing acceleration and reducing, the following expression for α_r is obtained:

$$\alpha_r = -\sum_{n=2}^{\infty} \sum_{m=0}^{n} \frac{\mu a^n}{r^{n+2}} A_{n+1,\ m+1} \left[R_m C_{n,m} + I_m S_{n,m} \right] \tag{2-30}$$

where

$$R_m = (1 - u^2)^{m/2} \cos m\lambda \tag{2-31}$$

$$I_m = (1 - u^2)^{m/2} \sin m\lambda \tag{2-32}$$

and where a recursion equation for $A_{n,m}$ is

$$A_{n+1,\ m+1} = uA_{n,m+1} + (n+m+1)\ A_{n,m} \tag{2-33}$$

The equations for the other α's (i.e. α_1, α_2, and α_3) are quite similar in form, varying principally in the subscripts used for A, R, and I.

An inspection of the above equations for the disturbing acceleration shows that the accuracy of the earth's non-spherical gravity model depends upon the number of C and S coefficient pairs that are incorporated. Likewise it is clear that uncertainties in the values used for the individual C and S coefficients will be reflected as uncertainties in the earth's disturbing acceleration due to the earth's non-spherical and non-symmetrical characteristics.

In the next section, data on uncertainties in these coefficients is presented. Also shown is the way in which these coefficients affect the propagation of the orbital ephemeris as reflected in landmark errors. In these studies it was assumed for convenience that the initial ephemeris errors were zero at the start of each observational pass over the USA.

2.6 LANDMARK ERROR CORRECTION TECHNIQUE

In the present study a relatively simple correction was applied to the position estimates of unknown landmarks observed during an observational pass over the USA. This correction was assumed to be that which is required to offset the discrepancy found between the estimated and true positions of a single control point (or known landmark) observed during the pass, usually at the start of the pass on the northern USA border. The discrepancy in the position estimate of the control point was due entirely to errors in the knowledge of satellite orbital position and attitude at the time of observation of the control point. If it were possible to keep the orbital position and attitude errors fixed during the observational pass, then the correction would be essentially the right one for all of the features observed during the pass. However, these errors do vary with time and some cause the correction to become less effective as the distance is increased between the control point and the other features. The correction applied to each unknown feature during an observational pass was that required to correct for the errors given for the control point in Equations 2-12 and 2-13.

It should be noted that all of the performance results in this report are based upon the assumption of only one control point per pass. By using additional control points it is obvious that a better landmark correction could be obtained. However, the present study was limited to a single control point for reasons of simplicity, and shows what the performance would be for this rather extreme case. In the performance results of Section 3, one can see some of the advantages to be gained by using additional well placed control points. However, no real consideration was given to multiple control points since this was considered to be more appropriate for future studies.

SECTION 3

PERFORMANCE RESULTS OF LANDMARK LOCATION STUDY

3.1 INTRODUCTION

The performance results in this report were primarily concerned with, first, the sensitivity of landmark location error to errors in each individual component of the orbital ephemeris and satellite attitude, and second, the combined effect of selected nominal error values for the components of the orbital ephemeris and satellite attitude. In addition, performance results are given showing the effect of incomplete gravitational modeling and of uncertainties in the gravitational coefficients. Performance results were also generated for various sets of errors obtained in the smoothed estimates of attitude of previous SIMS studies.

Most of the results were generated for an observational pass over the continental USA starting at 50° north latitude, 95° west longitude, and ending at 30° north latitude (see Figure 2-1). The time associated with the above latitude change was about 358 seconds. The orbital altitude was 540 nautical miles and the inclination was 99°. One exception to the mid-USA passes was for the gravitational modeling studies where the effect of variation in the initial longitude was investigated and found to be negligible for the intended application.

Unless otherwise stated, the true spacecraft attitude history was one where the roll and yaw angles were zero and the pitch angle changed at a constant orbital rate. In those cases where attitude librations (i.e. oscillations) were present, the attitude history was that given in Equation 2-17 of Section 2.3.

Unless specifically stated, the gyros were assumed to be error-free. When gyros errors were present, the nominal values used for each gyro were as follows:

True Bias Drift	0.03 deg/hr
Scale Factor Bias	10 PPM
Input Axis Misalignment (About each axis normal to gyro input axis)	10 arcsec
Random Drift (white noise (1σ))	0.01 deg/hr
Quantization (1σ)	0.1 arcsec

When gyro noises (i.e. random drift and quantization) were present, these noises were generated on a Monte Carlo basis. RMS data was obtained for only 20 computer runs and should therefore be considered as being representative.

Most of the results in this section give the data on landmark error relative to the ground track. The down-range component represents that component parallel to the ground track, and the cross-track component is that component normal to the ground track. (For brevity, these components are referred to as 'range' and 'track' in the tables.)

3.2 LANDMARK ERROR SENSITIVITY RESULTS

The performance results showing the sensitivity of landmark location error to error in each component of the orbital ephemeris and satellite attitude are given in Tables 3-1 through 3-17. Tables 3-1 through 3-9 give the sensitivity results for initial errors in spacecraft attitude and gyro bias drift, and Tables 3-10 through 3-17 give the sensitivity results for initial errors in the orbital ephemeris. In addition to the above sensitivity data, these tables also include data on the effects of:

1) Change in latitude
2) Change in scan beam angle (ϵ)
3) Spacecraft attitude librations
4) Gyro errors and noise

All of the sensitivity tables give the total (i.e. uncorrected) error in the determination of landmark position for a landmark observed at the start of the observational pass and for one or more landmarks observed later in the pass.* With the exception of Tables 3-7 and 3-16, which consider the effect of variation in the scan beam angle, ϵ, all of the landmarks were observed when $\epsilon = 0$ (i.e. downwards along the local vertical except for the cases with attitude libration in Tables 3-8 and 3-17). In all of the sensitivity results the location of the landmark observed at the start of the pass (i.e. at 50° latitude) was assumed to be known and was determined by using perfect knowledge of the orbital ephemeris and satellite attitude. The discrepancy between the known location and that computed with imperfect knowledge of orbital ephemeris and satellite attitude represents the landmark error given for that landmark in the tables. Using this discrepancy in the technique described in Section 2.6, a correction was generated for each landmark observed later in the observational pass. In the tables both the uncorrected and corrected errors are given for these later landmarks. The corrected data gives an indication of how well one can map one point with respect to another in the imagery.

In this report the known landmark is often referred to as the 'control point' in order to distinguish it from the other landmarks whose locations we wish to determine.

---

* It should be noted that all landmarks in this study were generated artificially as a matter of convenience.

3.2.1 LANDMARK ERROR SENSITIVITY TO INITIAL ERRORS IN SPACECRAFT ATTITUDE AND GYRO BIAS DRIFT

Tables 3-1 through 3-6 give the sensitivity of landmark errors to initial errors in pitch, roll, yaw, and the bias drift of each gyro. All of these tables give data for three error magnitudes in order to show how the landmark error varies with each error source. A fairly good check can be made on the data in Tables 3-1 through 3-6 by using the following equations for a near nominal attitude history:

$$\delta\theta_2 = \delta\theta_1 + \Delta t\, \delta b_y \tag{3-1}$$

$$\delta\phi_2 = c(\Delta\theta)\,\delta\phi_1 - s(\Delta\theta)\delta\psi_1 - \left[\frac{s(\Delta\theta)}{\omega}\right]\delta b_x - \left[\frac{1 - c(\Delta\theta)}{\omega}\right]\delta b_z \tag{3-2}$$

$$\delta\psi_2 = s(\Delta\theta)\delta\phi_1 + c(\Delta\theta)\,\delta\psi_1 - \left[\frac{1 - c(\Delta\theta)}{\omega}\right]\delta b_x + \left[\frac{s(\Delta\theta)}{\omega}\right]\delta b_z \tag{3-3}$$

where $\delta\theta_1, \delta\phi_1, \delta\psi_1, \delta b_x, \delta b_y$, and δb_z are the initial errors (at 50° latitude) in pitch, roll, yaw, and the bias drift of the roll, pitch, and yaw gyros, respectively; $\delta\theta_2, \delta\phi_2$, and $\delta\psi_2$ are the pitch, roll, and yaw errors at some later point in orbit; Δt is the time lapse between the two points in orbit; ω is the orbital rate; and $\Delta\theta$ is the angular change in orbital position (i.e. true anomaly). At a given point in orbit, the pitch, roll, and yaw errors $(\delta\theta_2, \delta\phi_2, \delta\psi_2)$ will cause the following landmark errors for a landmark observed near local vertical during a nominal attitude history:

$$\text{Down-Range Error} = K\left[-\delta\theta_2 + \delta\psi_2 \sin\epsilon\right] \tag{3-4}$$

$$\text{Cross-Track Error} = K\left[-\delta\phi_2\right] \tag{3-5}$$

where K is the distance subtended per unit angle (15.9 feet per arcsec at 540 nautical miles).

In the following subsections, frequent reference will be made to the above equations when discussing the effects of each error source. It should be noted that a quick comparison between the effects of the different error sources can be obtained by using Table 3-7.

3.2.1.1 Sensitivity to Pitch Error

In Table 3-1 it is seen that the landmark error caused by a given initial pitch error is independent of latitude change. This is due to the fact that the pitch error remains fixed throughout the orbit and causes a constant angular deviation of the downward pointing Z_B axis (see Figure 2-1) from the vertical in the orbital plane. Hence the resulting landmark error in range must remain constant. On the right side of Table 3-1 it is seen that the landmark error after correction for the initial error (using the known landmark at 50° latitude) is zero. This indicates that we can do a very good job of mapping one point with respect to another in the presence of pitch error.

3.2.1.2 Sensitivity to Roll Error

In Table 3-2 the effect of roll error on landmark error is initially the same in magnitude as that due to pitch error except that it is in track (see Equation 3-5). However, as the satellite moves ahead, the roll error changes in accordance with the first term on the right of Equation 3-2. The landmark error after correction for initial error increases approximately as $(1 - \cos \Delta\theta)$. In Table 3-2 it is seen that for small latitude changes, such as across the USA, the landmark error after correction is very small in comparison to the uncorrected error.

TABLE 3-1

SENSITIVITY OF LANDMARK ERRORS TO INITIAL PITCH ATTITUDE ERROR

Initial Pitch Error (arcsec)	Final Latitude (deg)	Landmark Error (feet) At Initial Latitude (50^o) Track	Landmark Error (feet) At Initial Latitude (50^o) Range	Landmark Error (feet) At Indicated Final Latitude Track	Landmark Error (feet) At Indicated Final Latitude Range	Landmark Error After Correction (feet) Track	Landmark Error After Correction (feet) Range
1	45	0	-15.9	0	-15.9	0	0
10			-159		-159		
50			-795		-795		
1	40		-15.9		-15.9		
10			-159		-159		
50			-795		-795		
1	30		-15.9		-15.9		
10			-159		-159		
50			-795		-795		
1	20		-15.9		-15.9		
10			-159		-159		
50			-795		-795		
1	10		-15.9		-15.9		
10			-159		-159		
50			-795		-795		
1	0		-15.9		-15.9		
10			-159		-159		
50			-795		-795		

Notes: 1) $\epsilon = 0$
2) No spacecraft librations

TABLE 3-2

SENSITIVITY OF LANDMARK ERRORS TO INITIAL ROLL ATTITUDE ERROR

Initial Roll Error (arcsec)	Final Latitude (deg)	Landmark Error (feet)				Landmark Error After Correction (feet)	
		At Initial Latitude (50^{o})		At Indicated Final Latitude			
		Track	Range	Track	Range	Track	Range
1	45	-15.9	0	-15.8	0	0.1	0
10	↓	-159	↓	-158	↓	1	↓
50	↓	-795	↓	-792	↓	3	↓
1	40	-15.9	↓	-15.7	↓	0.3	↓
10	↓	-159	↓	-157	↓	2	↓
50	↓	-795	↓	-782	↓	13	↓
1	30	-15.9	↓	-14.9	↓	1.0	↓
10	↓	-159	↓	-149	↓	10	↓
50	↓	-795	↓	-745	↓	50	↓
1	20	-15.9	↓	-13.7	↓	2.2	↓
10	↓	-159	↓	-137	↓	22	↓
50	↓	-795	↓	-685	↓	110	↓
1	10	-15.9	↓	-12.0	↓	3.9	↓
10	↓	-159	↓	-120	↓	39	↓
50	↓	-795	↓	-600	↓	195	↓
1	0	-15.9	↓	-10.0	↓	5.9	↓
10	↓	-159	↓	-100	↓	59	↓
50	↓	-795	↓	-500	↓	295	↓

Notes: 1) $\epsilon = 0$
2) No spacecraft librations

3.2.1.3 Sensitivity to Yaw Error

In Table 3-3 it is seen that an initial yaw error does not produce any error in the initial landmark (or control point) since the landmark is directly below the satellite (i.e. $\epsilon = 0$). However, as the satellite moves ahead, the initial error in yaw causes an error in roll as indicated by the second term on the right of Equation 3-2. The reason for this behavior is that the initial yaw error should really be looked upon as an error in inertial space. As the satellite's roll and yaw axes rotate in inertial space, this error is resolved into roll and yaw components as shown in Equations 3-2 and 3-3. A yaw error at one point in orbit becomes entirely a roll error after 90° of orbital motion. In Table 3-3 it is seen that for landmarks observed later in the pass, the error after correction is the same as before correction since no correction was called for in the control point.

In comparing the results of Tables 3-1 through 3-3, it is seen that yaw error has a much greater effect than pitch or roll error on the relative mapping capability.

3.2.1.4 Sensitivity to Gyro Bias Drift Error

The sensitivity of landmark errors to initial errors in the knowledge of bias drift of the roll (x), pitch (y), and yaw (y) gyros are shown in Tables 3-4 through 3-6. In each case there was no landmark error at the start of the observational pass since there was no error in spacecraft attitude at that point. However, as the satellite proceeds ahead, attitude errors will be introduced by the initial bias drift errors as shown in Equations 3-1 through 3-3. Since there was no error in the control point, there is no error correction for landmarks observed later in the pass.

In Table 3-4 the results are given for an initial error in roll (x) gyro bias drift. Here it is seen that the landmark error in track increases

TABLE 3-3

SENSITIVITY OF LANDMARK ERRORS TO INITIAL YAW ATTITUDE ERROR

Initial Yaw Error (arcsec)	Final Latitude (deg)	Landmark Error (feet) At Initial Latitude (50^o) Track	Landmark Error (feet) At Initial Latitude (50^o) Range	Landmark Error (feet) At Indicated Final Latitude Track	Landmark Error (feet) At Indicated Final Latitude Range	Landmark Error After Correction (feet) Track	Landmark Error After Correction (feet) Range
1	45	0	0	1.4	0	1.4	0
10				14		14	
50				70		70	
1	40			2.8		2.8	
10				28		28	
50				140		140	
1	30			5.6		5.6	
10				56		56	
50				280		280	
1	20			8.1		8.1	
10				81		81	
50				405		405	
1	10			10.4		10.4	
10				104		104	
50				520		520	
1	0			12.4		12.4	
10				124		124	
50				620		620	

Notes: 1) $\epsilon = 0$

2) No spacecraft librations

TABLE 3-4

SENSITIVITY OF LANDMARK ERRORS TO X GYRO BIAS DRIFT ESTIMATE ERROR

Initial b_x Error (deg/hr)	Final Latitude (deg)	Landmark Error (feet): At Initial Latitude (50^o) Track	Landmark Error (feet): At Initial Latitude (50^o) Range	Landmark Error (feet): At Indicated Final Latitude Track	Landmark Error (feet): At Indicated Final Latitude Range	Landmark Error after Correction (feet) Track	Landmark Error after Correction (feet) Range
.0015	45	0	0	2.1	0	2.1	0
.015				21		21	
.075				105		105	
.0015	40			4.2		4.2	
.015				42		42	
.075				210		210	
.0015	30			8.4		8.4	
.015				84		84	
.075				420		420	
.0015	20			12.2		12.2	
.015				122		122	
.075				610		610	
.0015	10			15.7		15.7	
.015				157		157	
.075				783		783	
.0015	0			18.6		18.6	
.015				186		186	
.075				931		931	

Notes: 1) $\epsilon = 0$
2) No spacecraft librations

TABLE 3-5

SENSITIVITY OF LANDMARK ERRORS TO Y GYRO BIAS DRIFT ESTIMATE ERROR

Initial b_y Error (deg/hr)	Final Latitude (deg)	Landmark Error (feet)				Landmark Error After Correction (feet)	
		At Initial Latitude (50^o)		At Indicated Final Latitude			
		Track	Range	Track	Range	Track	Range
.0015	45	0	0	0	-2.1	0	-2.1
.015					-21		-21
.075					-105		-105
.0015	40				-4.3		-4.3
.015					-43		-43
.075					-215		-215
.0015	30				-8.5		-8.5
.015					-85		-85
.075					-425		-425
.0015	20				-12.8		-12.8
.015					-128		-128
.075					-640		-640
.0015	10				-17.1		-17.1
.015					-171		-171
.075					-854		-854
.0015	0				-21.4		-21.4
.015					-214		-214
.075					-1068		-1068

Notes: 1) $\epsilon = 0$;
2) No spacecraft librations

TABLE 3-6

SENSITIVITY OF LANDMARK ERRORS TO Z GYRO BIAS DRIFT ESTIMATE ERRORS

Initial b_z Error (deg/hr)	Final Latitude (deg)	Landmark Error (feet)				Landmark Error After Correction (feet)	
		At Initial Latitude (50^{o})		At Indicated Final Latitude			
		Track	Range	Track	Range	Track	Range
.0015	45	0	0	0.1	0	0.1	0
.015				1.0		1.0	
.075				5		5	
.0015	40			0.4		0.4	
.015				4		4	
.075				20		20	
.0015	30			1.5		1.5	
.015				15		15	
.075				75		75	
.0015	20			3.4		3.4	
.015				34		34	
.075				168		168	
.0015	10			5.8		5.8	
.015				58		58	
.075				292		292	
.0015	0			8.9		8.9	
.015				89		89	
.075				446		446	

Notes: 1) $\epsilon = 0$
2) No spacecraft librations

almost linearly with latitude change angle for angles less than 30^{o}.

In Table 3-5 the results for an initial error in pitch (y) gyro bias drift are similar to those for the roll gyro in Table 3-4 except that the landmark error is now in range. It is seen that the landmark error varies almost linearly with latitude change angle. In this case, the slight departure from perfect linearity is due to the inclination of the orbit (99^{o}).

In Table 3-6 the sensitivity results are shown for initial yaw (z) gyro bias drift error. Here it is seen that the sensitivity is much smaller than that for the other two gyros.

3.2.1.5 Sensitivity to Attitude and Gyro Bias Drift Errors for Different Scan Beam Angles

Shown in Table 3-7 is the effect of attitude and gyro bias drift errors on landmark error for three different scan beam angles ($\epsilon = 0$, $\pm 5.3^{o}$). The results are for a complete pass over the USA (50^{o} to 30^{o} latitude) and show the errors in the control point on the northern border and in an unknown landmark on the southern border. The results for $\epsilon = 0$ were taken from the previous tables and provide a quick comparison of the relative sensitivities to attitude and gyro bias drift errors. It should be noted that the results given for each value of ϵ are for the case where both the control point and the unknown landmark were observed at that angle.

In comparing the results of Table 3-7 for the three values of ϵ it is seen that there is no change in the track component of the landmark error and very little change in the range component. Also note that the new range errors which appear in the right column of Table 3-7 for $\epsilon = \pm 5.3^{o}$ have equal but opposite polarities for the two values of ϵ.

Although no results were generated for the case where the scan

Table 3-7

SENSITIVITY OF LANDMARK ERRORS TO ERRORS IN SPACECRAFT ATTITUDE AND GYRO BIAS DRIFT FOR DIFFERENT SCAN BEAM ANGLES FOR 20° LATITUDE CHANGE

Initial Error	Landmark Error (feet)				Landmark Error After Correction (feet)	
	At Start (50° Lat)		At End (30° Lat)			
	Track	Range	Track	Range	Track	Range
For $\epsilon = 0^{\circ}$						
1 $\widehat{\sec}$ in Pitch	0	-15.9	0	-15.9	0	0
1 $\widehat{\sec}$ in Roll	-15.9	0	-14.9	0	1.0	0
1 $\widehat{\sec}$ in Yaw	0	0	5.6	0	5.6	0
.0015 deg/hr in B_x	0	0	8.4	0	8.4	0
.0015 deg/hr in B_y	0	0	0	-8.5	0	-8.5
.0015 deg/hr in B_z	0	0	1.5	0	1.5	0
For $\epsilon = +5.3^{\circ}$						
1 $\widehat{\sec}$ in Pitch	0	-15.9	0	-15.9	0	0
1 $\widehat{\sec}$ in Roll	-15.9	0	-14.9	0.5	1.0	0.5
1 $\widehat{\sec}$ in Yaw	0	1.5	5.6	1.4	5.6	-0.1
.0015 deg/hr in B_x	0	0	8.4	-0.1	8.4	-0.1
.0015 deg/hr in B_y	0	0	0	-8.5	0	-8.5
.0015 deg/hr in B_z	0	0	1.5	0.8	1.5	0.8
For $\epsilon = -5.3^{\circ}$						
1 $\widehat{\sec}$ in Pitch	0	-15.9	0	-15.9	0	0
1 $\widehat{\sec}$ in Roll	-15.9	0	-14.9	-0.5	1.0	-0.5
1 $\widehat{\sec}$ in Yaw	0	-1.5	5.6	-1.4	5.6	0.1
.0015 deg/hr in B_x	0	0	8.4	0.1	8.4	0.1
.0015 deg/hr in B_y	0	0	0	-8.5	0	-8.5
.0015 deg/hr in B_z	0	0	1.5	-0.8	1.5	-0.8

Note: Corrections assume that corrected data is for data point at same beam angle (ϵ) as control point or known landmark.

beam angle for the control point was different from that of the unknown landmark, one can see what the effect would be by analyzing the results in Table 3-7. For example, if $\epsilon = +5.3^{\circ}$ for the control point and $\epsilon = -5.3^{\circ}$ for the unknown landmark, the initial yaw error of one arcsec would cause a range error after correction of -2.9 feet instead of -0.1 feet. Note that the results for initial errors in pitch, roll, or gyro bias drift would be the same as given for $\epsilon = -5.3$ in Table 3-7 since these errors do not cause a change in the landmark error of the control point when ϵ is changed for that point.

3.2.1.6 Effect of Attitude Libration and Various Gyro Errors

In Table 3-8 it seems that spacecraft attitude librations have very little effect on the sensitivities to attitude and gyro bias drift errors.

Also shown in Table 3-8 are the results when gyro random drift and quantization noise were introduced. Although the results seem to be affected to some degree by gyro noise, it should be noted that the data happens to be that for one particular random run. A better indication of the effect of these noises is obtained by looking at Table 3-9 which gives the RMS data obtained from 20 Monte Carlo runs. Here it is seen that the nominal values of gyro random drift and quantization cause landmark errors on the order of 2-3 feet (1σ).

Also shown in Table 3-8 are the effects of gyro scale factor bias error and input axis misalignment in the presence of spacecraft attitude librations. It should be noted that in the absence of attitude librations these errors cause only an apparent fixed bias drift which cannot be distinguished from true bias drift. A detailed discussion of this phenomenon can be found in Section 5.5 of the Third Interim Technical Report[169]. When attitude librations are added to the nominal attitude history, the apparent bias drifts vary about their mean (or fixed) values. In Table 3-9 the performance results show only the effect of the variable portion of the apparent

Table 3-8

SENSITIVITY OF LANDMARK ERRORS TO ERRORS IN SPACECRAFT ATTITUDE AND GYRO BIAS DRIFT UNDER DIFFERENT CONDITIONS FOR 20° LATITUDE CHANGE

Initial Error	Landmark Error (feet)				Landmark Error After Correction (feet)	
	At Start (50° Lat)		At End (30° Lat)			
	Track	Range	Track	Range	Track	Range
No spacecraft librations						
1 $\widehat{sec}$ in Pitch	0	-15.9	0	-15.9	0	0
1 $\widehat{sec}$ in Roll	-15.9	0	-14.9	0	1.0	0
1 $\widehat{sec}$ in Yaw	0	0	5.6	0	5.6	0
.0015 deg/hr in B_x	0	0	8.4	0	8.4	0
.0015 deg/hr in B_y	0	0	0	-8.5	0	-8.5
.0015 deg/hr in B_z	0	0	1.5	0	1.5	0
With spacecraft librations						
1 $\widehat{sec}$ in Pitch	0	-15.9	0	-15.8	0	0.1
1 $\widehat{sec}$ in Roll	-15.9	0	-15.0	0.1	0.9	0.1
1 $\widehat{sec}$ in Yaw	0	0	5.3	0.1	5.3	0.1
.0015 deg/hr in B_x	0	0	8.4	0.1	8.4	0.1
.0015 deg/hr in B_y	0	0	0	-8.5	0	-8.5
.0015 deg/hr in B_z	0	0	1.5	0.1	1.5	0.1
With gyro drift noise and quantization noise, no librations						
1 $\widehat{sec}$ in Pitch	0	-15.9	5.0	-14.4	3.9	-0.8
1 $\widehat{sec}$ in Roll	-15.9	0	-12.8	0.7	1.0	1.7
1 $\widehat{sec}$ in Yaw	0	0	13.2	1.4	12.3	1.3
.0015 deg/hr in B_x	0	0	6.0	-5.1	5.2	-2.6
.0015 deg/hr in B_y	0	0	3.8	-6.6	4.0	-9.3
.0015 deg/hr in B_z	0	0	3.5	-5.5	3.1	-2.7

Notes: (1) ϵ = 0; (2) No gyro errors
(3) Run with gyro noise is for one particular random run

Table 3-9

SENSITIVITY OF LANDMARK ERRORS TO GYRO ERRORS AND NOISE FOR 20° LATITUDE CHANGE WITH ATTITUDE LIBRATIONS

Gyro Error or Noise	Error Value (1σ)	Landmark Error (feet)				Landmark Error After Correction (feet)	
		At start (50° Lat)		At end (30° Lat)			
		Track	Range	Track	Range	Track	Range
Scale Factor Bias Error	10 PPM	0	0	0.1	0.6	0.1	0.6
Input Axis Misalign	10 $\widehat{\text{sec}}$	0	0	-4.9	-2.4	-4.9	-2.4
Drift Rate White Noise	0.01 deg/hr	0	0	3.0	2.2	3.0	2.2
Quantization Noise	0.10 $\widehat{\text{sec}}$	1.5	2.0	2.2	2.3	1.5	1.6
Both Noises	-	1.5	2.0	4.0	2.5	3.5	2.3

Notes: (1) Error in estimate of average bias drift rate is assumed to be zero.

(2) Gyro errors and noise are assumed to apply equally to all 3 gyros.

(3) Landmark error data for gyro noises represents the rms for 20 random runs.

(4) $\epsilon = 0$

bias drift since the fixed component has the same effect as that given for gyro bias drift error in the preceding tables. Note also that the results are for only one point in the attitude libration history (i.e. $t \approx 358$ seconds).

3.2.2 LANDMARK ERROR SENSITIVITY TO INITIAL ERRORS IN ORBITAL EPHEMERIS

Tables 3-10 through 3-17 give the sensitivity of landmark errors to initial errors in orbital position and velocity. The magnitudes of the initial errors used to generate the results were 1, 3, and 6 times the nominal values which were 50 meters (164 feet) in down-range, 20 meters (66 feet) in altitude and cross-track, 0.05 meters per second (0.164 fps) in altitude rate, and 0.02 meters per second (0.066 fps) in down-range and cross-track rate. For brevity, the down-range and cross-track components of error are frequently referred to as 'range' and 'track'.

A fairly good check can be made on the data in Tables 3-10 through 3-16 by using the expressions for orbital position error in range (e_{RNG}), track (e_{TRK}), and altitude (e_{ALT}) in Equations 2-22 through 2-24. Note that for a nominal attitude history and $\epsilon = 0$, the orbital position errors e_{RNG} and e_{TRK} will also be the errors in landmark position. If $\epsilon \neq 0$, the altitude error will introduce a landmark error in track equal to $e_{ALT} \sin \epsilon$.

For relatively small changes in latitude, such as across the continental USA, the expressions in Equations 2-22 through 2-24 may be simplified to the following:

$$e_{ALT} = e_{ALT_0} \left[1 + (\omega t)^2/2\right] + e_{RNG_0} \left[\omega t\right] + \dot{e}_{ALT_0} \left[t\right] + \dot{e}_{RNG_0} \left[\omega t^2\right] \qquad (3\text{-}6)$$

$$e_{RNG} = -e_{ALT_0} \left[\omega t\right] + e_{RNG_0} \left[1 - (\omega t)^2\right] - \dot{e}_{ALT_0} \left[\omega t^2\right] + \dot{e}_{RNG_0} \left[t\right] \qquad (3\text{-}7)$$

$$e_{TRK} = e_{TRK_0}\left[1 - (\omega t)^2/2\right] + \dot{e}_{TRK_0}\left[t\right] \tag{3-8}$$

where the subscript 0 denotes the orbital ephemeris errors at the start of the observation pass (usually the northern USA border), t is the time since the start of the pass, and ω is the constant orbital rate. At a given point in orbit, the above errors will cause the following landmark errors:

$$\text{Down-Range Error} = e_{RNG} \tag{3-9}$$

$$\text{Cross-Track Error} = e_{TRK} + e_{ALT} \sin\epsilon \tag{3-10}$$

As previously indicated, the performance results in Tables 3-10 through 3-17 are for the case where a known landmark (or control point) is observed at the start of the observation pass (50° latitude), and unknown landmarks are observed later in the pass. All of the landmarks in Tables 3-10 through 3-16 were observed when the scan beam angle was zero. In Table 3-17 data is given for the case where $\epsilon \neq 0$.

3.2.2.1 Sensitivity to Altitude Error

In Table 3-10 the sensitivity to initial altitude error is shown for the case where $\epsilon = 0$. Note there is no error in the control point, and there is only an error in range for the unknown landmarks observed later in the pass. For small latitude changes this error varies almost linearly with latitude change angle and time (see first term on right of Equation 3-7).

3.2.2.2 Sensitivity to Track Error

Shown in Table 3-11 is the sensitivity to initial error in track. Note that the landmark error after correction is very small in comparison to the error before correction for landmarks in the USA. Also note that the landmark error after correction varies somewhat as the square of the latitude change angle.

TABLE 3-10

SENSITIVITY OF LANDMARK ERRORS TO INITIAL ALTITUDE ERROR

Initial Altitude Error (feet)	Final Latitude (deg)	Landmark Error (feet) At Initial Latitude (50°)		Landmark Error (feet) At Indicated Final Latitude		Landmark Error After Correction (feet)	
		Track	Range	Track	Range	Track	Range
66	45	0	0	0	-6	0	-6
198					-18		-18
396					-36		-36
66	40				-12		-12
198					-36		-36
396					-72		-72
66	30				-24		-24
198					-73		-73
396					-147		-147
66	20				-39		-39
198					-117		-117
396					-232		-232
66	10				-55		-55
198					-165		-165
396					-330		-330
66	0				-74		-74
198					-222		-222
396					-444		-444

Notes: 1) $\epsilon = 0$
2) No spacecraft librations

TABLE 3-11

SENSITIVITY OF LANDMARK ERRORS TO INITIAL TRACK ERROR

Initial Track Error (feet)	Final Latitude (deg)	Landmark Error (feet) At Initial Latitude (50°) Track	Landmark Error (feet) At Initial Latitude (50°) Range	Landmark Error (feet) At Indicated Final Latitude Track	Landmark Error (feet) At Indicated Final Latitude Range	Landmark Error After Correction (feet) Track	Landmark Error After Correction (feet) Range
66	45	66	0	65.7	0	-0.3	0
198		198		197		-1	
396		396		394		-2	
66	40	66		65		-1	
198		198		195		-3	
396		396		390		-6	
66	30	66		62		-4	
198		198		186		-12	
396		396		371		-25	
66	20	66		57		-9	
198		198		171		-27	
396		396		341		-55	
66	10	66		50		-16	
198		198		150		-48	
396		396		299		-97	
66	0	66		41		-25	
198		198		124		-74	
396		396		248		-148	

Notes: 1) $\epsilon = 0$
2) No spacecraft librations

3.2.2.3 Sensitivity to Range Error

The sensitivity of landmark errors to initial range errors is shown in Table 3-12. Here it is seen that the landmark error after correction is also small in comparison to that before correction for landmarks within the USA. Note that for small latitude changes, the landmark error after correction varies almost as the square of the latitude change.

At this point it would seem appropriate to comment on the definition of the range (or down-range) component of satellite position error used in this study. This error has been defined as the 'horizontal' down-range component of the satellite position error at a particular point in orbit, and not as the angular position error along the orbit (i.e. true anomaly error). This definition is frequently used and lends itself readily to the present treatment of satellite position and velocity in rectangular coordinates. Note in Equation 2-23 that for a given initial down-range error (e_{RNG_0}), the down-range error (e_{RNG}) at some other point will be different. However, if the initial down-range error had been defined to be an angular error in orbital position, the corresponding error at other points would have been the same and all landmarks (including the control point) would have been affected equally. In this case, the landmark error after correction would have been zero in Table 3-12.

3.2.2.4 Sensitivity to Orbital Velocity Errors

Shown in Tables 3-13 through 3-15 are the sensitivities of landmark errors to initial errors in altitude, track, and range rates. For small changes in latitude (i.e. with respect to 50° latitude), it is seen that there is less sensitivity to altitude rate error than to error in track or range rate. Also note that for small latitude change angles, the landmark error varies linearly with latitude change angle for initial errors in track or range rate, but varies as the square of the latitude change angle for initial errors in altitude rate (see also Equations 3-7 and 3-8).

TABLE 3-12

SENSITIVITY OF LANDMARK ERRORS TO INITIAL RANGE ERROR

Initial Range Error (feet)	Final Latitude (deg)	Landmark Error (feet) At Initial Latitude (50°) Track	Landmark Error (feet) At Initial Latitude (50°) Range	Landmark Error (feet) At Indicated Final Latitude Track	Landmark Error (feet) At Indicated Final Latitude Range	Landmark Error After Correction (feet) Track	Landmark Error After Correction (feet) Range
164	45	0	164	0	163	0	-1
492			492		488		-4
984			984		976		-8
164	40		164		159		-5
492			492		476		-16
984			984		953		-31
164	30		164		143		-21
492			492		430		-62
984			984		860		-124
164	20		164		118		-46
492			492		354		-138
984			984		708		-276
164	10		164		84		-80
492			492		252		-240
984			984		503		-481
164	0		164		42		-122
492			492		126		-366
984			984		250		-734

Notes: 1) $\epsilon = 0$
2) No spacecraft librations

TABLE 3-13

SENSITIVITY OF LANDMARK ERRORS TO INITIAL ALTITUDE RATE ERROR

Initial Altitude Error (feet/sec)	Final Latitude (deg)	Landmark Error (feet) At Initial Latitude (50°) Track		Landmark Error (feet) At Indicated Final Latitude Track		Landmark Error After Correction (feet) Track	
		Track	Range	Track	Range	Track	Range
.164	45	0	0	0	-1	0	-1
.492	↓	↓	↓	↓	-4	↓	-4
.984					-8		-8
.164	40				-5		-5
.492	↓				-16		-16
.984					-31		-31
.164	30				-21		-21
.492	↓				-62		-62
.984					-124		-124
.164	20				-46		-46
.492	↓				-138		-138
.984					-276		-276
.164	10				-80		-80
.492	↓				-241		-241
.984					-482		-482
.164	0				-123		-123
.492	↓				-368		-368
.984					-735		-735

Notes: 1) $\epsilon = 0$
2) No spacecraft librations

TABLE 3-14

SENSITIVITY OF LANDMARK ERRORS TO INITIAL TRACK RATE ERROR

Initial Track Rate Error (ft/sec)	Final Latitude (deg)	Landmark Error (feet)				Landmark Error After Correction (feet)	
		At Initial Latitude (50^{o})		At Indicated Final Latitude			
		Track	Range	Track	Range	Track	Range
.066	45	0	0	6	0	6	0
.198				18		18	
.396				35		35	
.066	40			12		12	
.198				35		35	
.396				70		70	
.066	30			23		23	
.198				69		69	
.396				139		139	
.066	20			34		34	
.198				102		102	
.396				203		203	
.066	10			43		43	
.198				129		129	
.396				260		260	
.066	0			51		51	
.198				154		154	
.396				309		309	

Notes: 1) $\epsilon = 0$
2) No spacecraft librations

TABLE 3-15

SENSITIVITY OF LANDMARK ERRORS TO INITIAL RANGE RATE ERROR

Initial Range Rate Error (ft/sec)	Final Latitude (deg)	Landmark Error (feet) At Initial Latitude (50°) Track	Landmark Error (feet) At Initial Latitude (50°) Range	Landmark Error (feet) At Indicated Final Latitude Track	Landmark Error (feet) At Indicated Final Latitude Range	Landmark Error After Correction (feet) Track	Landmark Error After Correction (feet) Range
.066	45	0	0	0	6	0	6
.198					18		18
.396					35		35
.066	40				12		12
.198					35		35
.396					69		69
.066	30				22		22
.198					65		65
.396					130		130
.066	20				29		29
.198					87		87
.396					173		173
.066	10				31		31
.198					94		94
.396					190		190
.066	0				29		29
.198					87		87
.396					173		173

Notes: 1) $\epsilon = 0$
2) No spacecraft librations

3.2.2.5 Sensitivity to Orbital Ephemeris Errors for Different Scan Beam Angles

Shown in Table 3-16 is the effect of orbital ephemeris error on landmark error for three different scan beam angles ($\epsilon = 0, \pm 5.3^\circ$). The results are for a complete pass over the USA (50° to 30° latitude) and show the landmark errors of the control point on the northern border and of the unknown landmark on the southern border.

The data for each value of ϵ in Table 3-16 is for the case where both the control point and the unknown landmark were observed at that angle. The results for $\epsilon = 0$ were taken from the previous tables and provide a quick comparison of the relative sensitivities of the different ephemeris errors.

In comparing the results for the three values of ϵ it is seen that a change in ϵ affects the landmark error of the control point only in the presence of an altitude error. Consequently, the landmark correction, which uses the error of the control point, is influenced by a change in ϵ only in the presence of an altitude error. For all cases, note that the range component of the landmark error after correction is not affected by a change in ϵ. (The opposite was true in the presence of spacecraft attitude and gyro bias drift errors). Most of the corrected landmark track errors which appear when ϵ is changed from 0° to $\pm 5.3^\circ$ are due to the change in ϵ only for the unknown landmark.

Although no results were generated for the case where the scan beam angle for the control point was different from that of the unknown landmark, one can see what the effect would be by analyzing the results in Table 3-16. The only initial ephemeris error requiring consideration is the altitude error since the error of the control point is not affected by a change in ϵ when the other ephemeris errors are present. For the case of initial altitude error:

Table 3-16

SENSITIVITY OF LANDMARK ERRORS TO NOMINAL EPHEMERIS ERRORS FOR DIFFERENT SCAN BEAM ANGLES FOR 20° LATITUDE CHANGE

Initial Error	Landmark Error (feet)				Landmark Error After Correction (feet)	
	At start (50° Lat)		At end (30° Lat)			
	Track	Range	Track	Range	Track	Range
For $\epsilon = 0^\circ$						
66 ft. in Altitude	0	0	0	-24	0	-24
66 ft. in Track	66	0	62	0	-4	0
164 ft. in Range	0	164	0	143	0	-21
.164 fps in Alt. Rate	0	0	0	-21	0	-21
.066 fps in Track Rate	0	0	23	0	23	0
.066 fps in Range Rate	0	0	0	22	0	22
For $\epsilon = +5.3^\circ$						
66 ft. in Altitude	6.1	0	6.5	-24	0.4	-24
66 ft. in Track	66	0	62	0	-4	0
164 ft. in Range	0	164	5	143	5	-21
.164 fps in Alt. Rate	0	0	5	-21	5	-21
.066 fps in Track Rate	0	0	23	0	23	0
.066 fps in Range Rate	0	0	1	22	1	22
For $\epsilon = -5.3^\circ$						
66 ft. in Altitude	-6.1	0	-6.5	-24	-0.4	-24
66 ft. in Track	66	0	62	0	-4	0
164 ft. in Range	0	164	-5	143	-5	-21
.164 fps in Alt. Rate	0	0	-5	-21	-5	-21
.066 fps in Track Rate	0	0	23	0	23	0
.066 fps in Range Rate	0	0	-1	22	-1	22

Note: No spacecraft librations

If $\epsilon = +5.3^{\circ}$ for the control point and $\epsilon = -5.3^{\circ}$ for the unknown landmark, an initial altitude error of 66 feet will cause a track error after correction of -12.6 feet instead of -0.4 feet.

3.2.2.6 Effect of Attitude Libration and Gyro Noise

In Table 3-17 it is seen that spacecraft attitude librations have very little effect on the sensitivities to orbital ephemeris errors. Also shown in Table 3-17 are the results when gyro random drift and quantization noise were present. It should be noted, however, that this is only for one particular random run, and that a better indication of the effect of gyro noise can be obtained by referring to Table 3-9 which gives the RMS data for 20 Monte Carlo runs.

3.3 EFFECT OF HIGHER HARMONICS IN EARTH'S GRAVITATIONAL FIELD

In this section data is given on the effects of the higher harmonic coefficients of the Earth's gravitational field. The primary purpose of this effort was to generate data indicating the accuracy required in a model of the Earth's gravitational field in order to achieve a given accuracy in landmark location determination in the continental USA. As usual, the satellite orbit was initially assumed to be circular with an altitude of 540 nautical miles and an inclination of 99 degrees.

3.3.1 GRAVITATIONAL MODEL OF EARTH

In Section 2.5 some details were given on the gravitational model of the Earth. The reference model used in this study was the 1967 Smithsonian gravitational model, for which the nominal values of all harmonic coefficients up to $C_{6,6}$ and $S_{6,6}$ are given in Table 3-18. The table also compares the 1967 values with those from other sources, including the more recent 1969 Smithsonian model. References 179 through 182 were used to obtain relevant data.

Table 3-17

SENSITIVITY OF LANDMARK ERRORS TO NOMINAL EPHEMERIS ERRORS UNDER DIFFERENT CONDITIONS FOR 20° LATITUDE CHANGE

Initial Error	Landmark Error (feet)				Landmark Error After Correction (feet)	
	At start (50° Lat)		At end (30° Lat)			
	Track	Range	Track	Range	Track	Range
No spacecraft librations						
66 ft. in Altitude	0	0	0	-24	0	-24
66 ft. in Track	66	0	62	0	-4	0
164 ft. in Range	0	164	0	143	0	-21
.164 fps in Alt. Rate	0	0	0	-21	0	-21
.066 fps in Track Rate	0	0	23	0	23	0
.066 fps in Range Rate	0	0	0	22	0	22
With spacecraft librations						
66 ft. in Altitude	0	-0.6	-0.1	-23	-0.1	-22.4
66 ft. in Track	66	0	62	0.1	-4	0.1
164 ft. in Range	0	164	-0.1	144	-0.1	-20
.164 fps in Alt. Rate	0	0	-0.1	-20	-0.1	-20
.066 fps in Track Rate	0	0	23	0.1	23	0.1
.066 fps in Range Rate	0	0	0	22	0	22
With gyro drift noise and quantization noise, no libration						
66 ft. in Altitude	1.1	2.3	5.0	-22.9	3.9	-25.2
66 ft. in Track	68	-1.0	64	0.7	-4	1.7
164 ft. in Range	0.8	164	7.6	145	6.8	-19
.164 fps in Alt. Rate	0.8	-2.5	-2.4	-26	-3.2	-23.5
.066 fps in Track Rate	-0.2	2.7	27	1.9	27.2	-0.9
.066 fps in Range Rate	0.4	-2.7	2.0	16	1.6	18.7

Notes: (1) $\epsilon = 0$, (2) No gyro errors
(3) Run with gyro noise is for a particular random run

Table 3-18

NON-SPHERICAL HARMONIC COEFFICIENTS FOR THE GEOPOTENTIAL AND THEIR UNCERTAINTIES

Coefficient	Coefficients of 1967 Model (Normalized values x 10^{-6})		Change From '67 to '69 Model	Spread in Values as Computed by Other Observers			Assumed Uncertainty (1σ)
n, m	C	S	(percent)	(percent)			(percent)
2, 0	-1082.639	0	0.001				0.0012
2, 2	2.38	-1.35	1.3				2
3, 0	2.565	0	-1.1				2
3, 1	1.71	0.23	16				10
3, 2	0.84	-0.51	11	-7	to	+10	10
3, 3	0.66	1.43	1.3				3
4, 0	1.608	0	-1.0				2
4, 1	-0.47	-0.39	18	-8	to	+28	30
4, 2	0.35	0.48	32	0	to	+32	30
4, 3	0.92	-0.24	5	-13	to	+13	30
4, 4	0.04	0.30	17	-27	to	+77	30
5, 0	0.174	0	32				30
5, 1	-0.06	-0.05	41	-38	to	+163	40
5, 2	0.53	-0.21	23	-25	to	+33	40
5, 3	-0.40	0.07	7	-68	to	+51	40
5, 4	-0.20	0.02	35	-40	to	+185	40
5, 5	0.18	-0.56	5	-15	to	+14	40
6, 0	-0.542	0	7	-20	to	+39	40
6, 1	-0.08	0.01	25	-38	to	+163	40
6, 2	0.01	-0.27	30	-7	to	+74	40
6, 3	-0.04	0.03	.10	-40	to	+160	40
6, 4	-0.08	-0.48	-17	-23	to	+23	40
6, 5	-0.26	-0.46	6	-28	to	+47	40
6, 6	-0.02	-0.16	-25	0	to	+200	40

The uncertainties assumed for the coefficients are given in the last column of Table 3-18. Due to difficulty in obtaining sufficient data to firmly establish the uncertainties of these coefficients, the following procedure was used to generate the assumed values: One was to note the changes in the values of the C coefficients for the 1967 and 1969 Smithsonian models and assume that a similar change is probable in the future. The other was to note the spread in the values of the coefficients reported by other observers. The assumed uncertainties listed in Table 3-18 are probably on the pessimistic side. Of particular interest in this table is the percentage uncertainty of $C_{2,0}$ relative to that of $C_{2,2}$. This shows how well the most important coefficient $C_{2,0}$ (frequently denoted as J_2) has been determined.

3.3.2 SENSITIVITY TO HARMONIC COEFFICIENTS

Table 3-19 gives the effect on landmark location determination of each pair of C and S harmonic coefficients for an observational pass over the middle of the continental USA (50° to 30° latitude). The initial error in orbital position and velocity at 50° latitude was zero. The results in Table 3-19 represent the errors which would occur in a landmark on the southern border if the indicated coefficient pair was not accounted for in the propagation of satellite position and velocity. It is important to note that this table gives the effect on landmark location due to the coefficient pairs themselves and not to errors (i.e. uncertainties) in the coefficients. It is seen that the landmark error due to $C_{2,0}$ (i.e. J_2) predominates by far that due to any other coefficient pair. The next largest error (8.1 feet) is that due to the 2, 2 pair. The remaining n, m pairs which cause an error of at least 3 feet are (3, 0), (3, 1), (3, 2), (3, 3), (4, 3), and (6, 4).

In Table 3-20 the landmark error is shown for various combinations of coefficient pairs. The most important combination (or model) is probably the one containing all coefficients through $C_{6,6}$ and $S_{6,6}$ (except for J_2). It is seen that the omission of this coefficient combination in satellite ephemeris propagation will result in a landmark error of only 8.5 feet for a complete

Table 3-19

SENSITIVITY OF LANDMARK ERRORS TO INDIVIDUAL GRAVITATIONAL HARMONIC C AND S COEFFICIENTS FOR PASS OVER USA

Coefficient	Landmark Error (feet)			Coefficient	Landmark Error (feet)		
n, m	Track	Range	RSS	n, m	Track	Range	RSS
2, 0	397	-1, 503	1, 555	6, 2	0. 1	-0. 4	0. 4
3, 0	-0. 8	3. 8	3. 9	6, 3	0. 2	0	0. 2
4, 0	-0. 1	1. 1	1. 1	6, 4	-3. 1	-1. 1	3. 3
5, 0	0	0	0	6, 5	0. 6	-1. 7	1. 8
6, 0	-0. 1	0. 5	0. 5	6, 6	0. 2	0. 2	0. 3
7, 0	-0. 1	-0. 4	0. 4	7, 1	-0. 3	0. 3	0. 4
8, 0	0. 0	-0. 1	0. 1	7, 2	0. 7	-1. 3	1. 5
9, 0	0. 0	0	0	7, 3	-0. 4	0. 3	0. 5
10, 0	-0. 1	0. 2	0. 2	7, 4	-0. 6	0	0. 6
2, 2	1. 0	8. 0	8. 1	7, 5	0. 4	0. 2	0. 4
3, 1	3. 9	-0. 3	3. 9	7, 6	0. 5	-0. 6	0. 8
3, 2	1. 0	3. 1	3. 3	7, 7	0	-0. 1	0. 1
3, 3	0. 5	-4. 4	4. 4	8, 1	0	0	0
4, 1	-0. 8	2. 5	2. 6	8, 2	0. 1	0	0. 1
4, 2	-2. 4	-0. 4	2. 4	8, 3	0	-0. 1	0. 1
4, 3	-5. 2	-1. 5	5. 4	8, 4	-0. 2	-0. 4	0. 4
4, 4	0. 7	0. 4	0. 8	8, 5	-0. 9	-0. 1	0. 9
5, 1	0	0. 3	0. 3	8, 6	-1. 2	-1. 0	1. 6
5, 2	0. 9	-2. 1	2. 3	8, 7	0. 1	-0. 1	0. 1
5, 3	2. 4	0. 6	2. 5	8, 8	0. 2	0	0. 2
5, 4	-0. 5	0. 6	0. 8	9, 5	-0. 3	0	0. 3
5, 5	1. 2	-0. 4	1. 3	9, 8	0. 3	-0. 1	0. 3
6, 1	0. 1	0	0. 1	10, 7	-0. 4	-0. 1	0. 4

Note: Only the most significant coefficients among the $C_{9, m}$ and $C_{10, m}$ coefficients are listed.

Table 3-20

LANDMARK ERROR VS. CHANGE IN LATITUDE FOR SEVERAL NON-SPHERICAL GRAVITY HARMONIC MODELS

Latitude (deg)	Lat. change Angle (deg)	Landmark Error (feet) Due to all coeffs. thru $C_{6,6}$, $S_{6,6}$ (except J_2)	Due to $C_{2,2}$, $S_{2,2}$ coefficients only	Due to 0.001 J_2 coefficient only
50	0	0.0	0.0	0.0
40	10	2.4	1.7	0.4
30	20	8.5	7.7	1.5
20	30	17.8	18.9	3.0
10	40	31.4	36.2	4.6
0	50	51.8	60.3	6.1
-10	60	81.4	91.4	7.1
-20	70	121.3	129.7	7.4
-30	80	171.8	175.1	6.6
-40	90	233.3	227.5	4.7

Note: 0.001 J_2 is approximately 100 x nominal uncertainty of J_2

pass over the USA. It is also seen that the coefficient pair ($C_{2,2}$ and $S_{2,2}$) is the major contributor in the previous combination. In the right column of Table 3-20 the landmark error is shown for 0.001 J_2, which is about 100 times the nominal uncertainty of J_2. This emphasizes how insignificant the effect of the J_2 uncertainty is on landmark error.

3.3.3 SENSITIVITY TO UNCERTAINTIES IN HARMONIC COEFFICIENTS

Having presented in Section 3.3.2 the effect of nominal C and S coefficients on landmark location, we now consider the effect of uncertainties in these coefficients. In Table 3-21 the effect of coefficient uncertainties is shown for various coefficient combinations. To obtain a better indication of performance for certain combinations, the RMS value of the landmark error was determined for 25 runs in which the uncertainties were randomized on a one-sigma basis. Most significant in Table 3-21 are the results obtained for all coefficients up to $C_{6,6}$ and $S_{6,6}$. The RSS landmark error was 1.7 feet, which is smaller than that obtained with the Monte Carlo runs for gyro drift and quantization noise in Table 3-9. Also of interest in Table 3-21 is the fact that if the uncertainties for all C and S coefficients (except J_2) were 30% of the nominal values of the coefficients, the resulting RSS landmark error is 4.5 feet, which is not much greater than 1.7 feet.

To summarize, it is clear from the preceding results that the effect of gravitational uncertainties on landmark error is insignificant. For this reason, the combined error studies reported on in Section 3.5 did not include the effects of gravitational uncertainties.

3.4 EFFECT OF ERRORS IN THE SMOOTHED ESTIMATES OF SPACECRAFT ATTITUDE AND GYRO BIAS DRIFT

The data in this section attempts to show the landmark errors produced by the errors in some of the smoothed estimates of spacecraft

Table 3-21

EFFECT OF UNCERTAINTIES IN NON-SPHERICAL GRAVITATIONAL HARMONIC COEFFICIENTS ON LANDMARK ERROR

UNCERTAINTY in FOLLOWING	Monte Carlo Run?	Landmark Error (feet)		
		Track	Range	RSS
$C_{2,0}$ (J_2) only	No	0.01	0.02	0.02
$C_{2,2}$; $S_{2,2}$ only	No	0.02	0.18	0.18
All coefficients through $C_{3,3}$; $S_{3,3}$	Yes	0.45	0.51	0.7
All coefficients through $C_{6,6}$; $S_{6,6}$	Yes	1.4	1.0	1.7
All coefficients where the uncertainties are assumed to be 30% of nominal values of the coefficients (J_2 not included)	Yes	2.9	3.5	4.5

Note RMS results for 25 runs are given for all Monte Carlo runs.

attitude and gyro bias drift of previous SIMS studies. Presented in Table 3-22 are many sets of errors in the smoothed estimates of spacecraft attitude and gyro bias drift. These are the errors which were present at the start of the observational pass over the USA. Most of the error sets represent the errors given in the Final Report[177] for SIMS-A configurations with different star mapper measurement errors and fields-of-view (FOV), and different data processing intervals. The error sets represent sample errors since they were obtained for particular sequences of random star mapper measurement error, random gyro drift, etc. Consequently, these error sets are not a statistical indication of performance of the SIMS-A configurations. To obtain a statistical indication of performance one must use the uncertainties given for the smoothed estimates in the Final Report[177]. There it will be found that the yaw attitude uncertainty for most of the SIMS-A configurations exceeds 3.6 arcsecs (0.001 degree), which was the requirement for each axis in the original SIMS studies.

The landmark errors in Table 3-22 represent the RSS values of the corrected landmark errors for an unknown landmark observed on the southern border (30^{o} latitude). The control point (or known landmark) was observed on the northern border. It is interesting to note that the landmark errors for the various SIMS-A configurations are almost always less than 10 feet. This represents one of the most significant results in the present effort since it indicates that for relative mapping, we are not as sensitive to errors in the smoothed estimates of pitch, roll, yaw, and gyro bias drift as was indicated for the individual errors in Section 3.2.1. This is due to the existence of strong negative correlations between certain errors in the smoothed estimates of attitude and gyro bias drift. The result is a partial cancellation of the effects of certain errors on the landmark location problem. Inspection of previous SIMS-A and SIMS-B data on the smoothed covariances of attitude and bias drift uncertainties indicate that the correlation between certain errors approaches -1 after a certain amount of smoothing. Evidence of such correlation can be seen in Table 3-22 by comparing the SIMS-A yaw errors with the corresponding roll bias drift errors (b_x). Note that the magnitude of the yaw error in

Table 3-22

ERRORS IN SMOOTHED ESTIMATES OF ATTITUDE AND BIAS DRIFT AND RESULTING LANDMARK ERRORS FOR DIFFERENT SIMS CONFIGURATIONS

Case	Mapper FOV (deg)	Data Interval (orbits)	Estimate Error in						RSS Ldmk. Error After Correct. (feet)
			Attitude (arcsec)			Bias Drift (10^{-3} deg/hr)			
			Pitch	Roll	Yaw	b_x	b_y	b_z	
SIMS-A ERRORS									
A	4	1	-0.4	0.2	4.1	-5.5	.15	-.60	8.2
A		2	0.4	-0.8	1.6	-1.4	.15	0	1.0
A		4	-0.3	1.0	1.2	-0.9	.30	-.15	3.0
B		1	0.4	1.8	-15.1	13.8	-.30	-.75	6.2
C		1	0.6	-1.5	- 7.9	7.1	-.45	.60	6.1
D		1	0	-0.7	5.8	-3.9	.15	1.05	10.8
D		2	0.4	-0.5	- 0.8	1.1	.30	-.15	1.7
E		1	-0.1	0.2	12.3	-12.0	.45	1.05	3.9
E		2	-0.3	0.7	3.2	- 2.9	.30	.15	3.2
F	10	1	1.1	0.5	7.9	- 8.4	.30	-1.05	3.8
F	10	4	-0.1	-0.7	1.2	- 2.0	.15	-.30	5.3
G	4	2	1.9	-3.4	16.5	-14.7	.30	.60	7.2
G	4	6	1.0	1.3	15.9	-16.7	.15	-1.35	4.5
H	10	1	3.3	3.1	22.9	-28.5	.75	-3.8	32.5
H	10	4	-0.1	-1.0	9.2	- 9.2	.15	-0.6	1.7
SIMS-B TYPICAL ERRORS (see text)									
-	-	1	-0.6	-0.4	1.1	-0.5	-.15	-.15	3.3
SIMS-L TYPICAL ERRORS (see text)									
-	-	-	2.2	2.2	23.8	-23.7	.45	29.7	60

Notes: Cases B, C - different mapper and gyro noise sequences used than for A.

Case D - no spacecraft librations; otherwise librations are present.

Case E - large initial attitude estimation errors

SIMS-A - Star Mapper Error (1σ): this is 1.1 arcsec for Cases A through E. It is 2 arcsecs for Case F and 8 arcsecs for Cases G & H.

arcsecs is almost the same as the roll bias drift error in units of 10^{-3} degrees per hour (or 10^{-3} arcsec per second). Also note the consistent reversal in polarity of these two errors.

A fairly good check can be made on the RSS landmark errors in Table 3-22 as follows: Using Equations 3-1 through 3-5, the down-range and cross-track errors after correction can be expressed as follows for the case of $\epsilon = 0$:

$$\text{Corrected Down-Range Error} = -K\left[\delta\theta_2 - \delta\theta_1\right] = -K\Delta t\,\delta b_y \qquad (3\text{-}11)$$

$$\text{Corrected Cross-Track Error} = -K\left[\delta\phi_2 - \delta\phi_1\right]$$

$$= -K\left[c(\Delta\theta) - 1\right]\delta\phi_1 - s(\Delta\theta)\,\delta\psi_1 - \left[\frac{s(\Delta\theta)}{\omega}\right]\delta b_x - \left[\frac{1 - c(\Delta\theta)}{\omega}\right]\delta b_z \qquad (3\text{-}12)$$

The RSS of the above errors represents the desired check on the results in the right column of Table 3-22.

In Equation 3-12 it is also interesting to compare the terms containing the yaw error ($\delta\psi_1$) and the roll bias drift error (δb_x). For the present orbit, $\omega \approx 10^{-3}$ radians per second. Consequently, these two terms will cancel each other out if $\delta\psi_1$ (in arcsecs) $= -\delta b_x$ (in 10^{-3} arcsecs per second), as is almost the case for most of the SIMS-A errors in Table 3-22.

The data presented in Table 3-23 shows how the landmark error varies for different conditions, given a particular set of errors in the smoothed estimates of attitude and gyro bias drift. This error set was taken from Table 3-22 and represents one of the worst SIMS-A configurations studied in the past. Table 3-23 shows that attitude librations do not have any significant effect on performance except in the presence of certain gyro errors.

Table 3-23

EFFECT OF A SET OF SIMS-A SMOOTHED ESTIMATE ERRORS ON LANDMARK ERRORS UNDER DIFFERENT CONDITIONS

Parameters			Landmark Error (feet)				Landmark Error after Correction (ft.)	
			At Initial Latitude (50^o)		At Final Latitude (30^o)			
Librations ?	Gyro Drift & Quant. Noise ?	Gyro Bias, Scale Factor Error & IA Misalignment ?	Track	Range	Track	Range	Track	Range
No	No	No	10.3	0.5	21.1	-0.4	10.7	-0.9
Yes	No	No	10.3	0.5	19.4	-0.4	9.1	-0.9
No	Yes	No	10.4	1.6	23.2	3.5	13.0	3.0
No	No	Yes	10.3	0.5	21.1	-0.4	10.7	-0.9
Yes	No	Yes	10.3	0.5	14.5	-2.2	4.2	-2.6
No	Yes	Yes	10.4	1.6	23.2	3.5	13.0	3.0
Yes	Yes	Yes	10.4	1.6	16.7	4.1	6.7	3.9

Notes: (1) Data above obtained with following SIMS-A error set:
[-.03, -.7, 5.8; -.0039, .00015, .00105] arcsec, deg/hr
These smoothed estimate errors were obtained for the midpoint of a 1 orbit data interval with 1.1 arcsec mapper sensor error and 4^o optics FOV.

(2) Scan beam oriented downwards, $\epsilon = 0$.

(3) No initial navigation errors.

(4) When gyro noise is present, data given is for the rms of 25 Monte Carlo runs.

3.5 COMBINED EFFECT OF ERRORS IN SPACECRAFT ATTITUDE AND ORBITAL EPHEMERIS

Until now, consideration has only been given to the separate effects of errors in spacecraft attitude and orbital ephemeris. In this section, these errors are combined to determine the overall effect on landmark error. No consideration was given to gravitational coefficient uncertainties since their effects were found to be relatively small for observational passes over the continental USA. Although the data presented in this section may not give as much insight into the effects of individual error sources, it has been generated as a matter of interest.

To implement this phase of the study, the following errors were adopted as nominal for spacecraft attitude and gyro bias drift at the start of the observational pass:

$$[\,2,\ 2,\ 4;\ -.0030,\ -.0015,\ .0015\,]\ \text{arcsec, deg/hr}$$

which are the respective errors in pitch, roll, yaw, and the bias drift of the X, Y, and Z gyros. The 4 arcsec error in yaw approximately corresponds to the 0.001 degree error originally specified for the SIMS trade study. The reason for the value of 2 arcsecs in pitch and roll is simply a recognition of the better performance observed for these two angles in the previous SIMS studies. The gyro bias drift errors were chosen to be compatible with the attitude errors in accordance with the smoothed results of previous studies.

The nominal errors adopted for the orbital ephemeris were the same as those given in Section 2.4. As previously indicated in Section 2.4, the data used to establish these values was somewhat limited but was considered to be sufficient for most of the studies. However, in the present case, it would have been nice to have had one or more sets of errors which are representative of the present ground tracking and batch-processing techniques. It is felt that this data would reflect the correlation between certain ephemeris errors, just as was the case for the smoothed estimates of spacecraft attitude and gyro bias drift in the SIMS studies. It would also have been desirable to have one or more covariance matrices of the ephemeris errors so that randomized sets of errors could be generated. In the present effort, the nominal values selected for the ephemeris errors were:

$$[\,-20,\ 20,\ 50;\ -.05,\ .02,.02\,]\ \text{meters, meters/sec}$$

which are respectively the errors in altitude, track, range, altitude rate, track rate, and range rate. The relative magnitudes and polarities of certain errors were adopted to reflect certain error relationships which have been encountered in other studies at CSDL.

3.5.1 COMBINED EFFECT OF NOMINAL ATTITUDE AND EPHEMERIS ERRORS

Tables 3-24 through 3-27 and Figures 3-1 through 3-5 present data on the separate and combined effects of nominal errors in the ephemeris and of nominal errors in spacecraft attitude and gyro bias drift. Table 3-24 gives data on the uncorrected landmark errors versus latitude for an observational pass over the USA. This data is illustrated graphically in Figures 3-1 and 3-2. It is seen that the nominal ephemeris errors make a greater contribution to the uncorrected landmark errors than the nominal errors of attitude and gyro bias drift errors. Shown also is the effect of different scan beam angles. Note in Table 3-24 and Figure 3-1 that a variation of the scan beam angle, ϵ, has almost no effect on the down-range component of landmark error when only ephemeris errors are present, and that the same is true for the cross-track component of landmark error when only attitude and gyro bias drift errors are present. Thus when we inspect the results for the combined error sources in Figure 3-2, we know from the above that the spread in the down-range error curves for different scan beam angles is due to the attitude and gyro bias drift errors; while the spread in the cross-track error curves is due

Table 3-24

UNCORRECTED LANDMARK ERRORS VS. LATITUDE DUE TO THE PRINCIPAL NOMINAL ERROR SOURCES FOR DIFFERENT SCAN BEAM ANGLES

Latitude (deg)	Landmark Errors in Feet Due to Nominal Initial Errors In					
	Ephemeris		Attitude and Bias Drift		Both Cases Combined	
	Track	Range	Track	Range	Track	Range
For $\epsilon = 0$						
50	65.6	164.6	-31.8	-31.8	34.9	135.1
45	71.7	175.9	-30.6	-34.4	43.9	142.4
40	76.9	187.1	-29.1	-36.7	48.7	151.5
35	81.2	198.2	-27.1	-38.7	55.1	159.9
30	84.6	209.4	-24.2	-40.4	60.7	168.4
For $\epsilon = +5.3^{\circ}$						
50	59.6	164.6	-32.0	-25.9	28.7	141.1
45	65.7	176.0	-30.8	-28.0	37.7	149.1
40	71.0	187.1	-29.3	-30.0	42.6	158.9
35	75.4	198.2	-27.3	-31.5	49.1	168.0
30	79.0	209.3	-24.5	-32.8	54.8	176.9
For $\epsilon = -5.3^{\circ}$						
50	71.8	164.5	-32.2	-37.7	40.7	129.1
45	77.8	175.9	-31.0	-40.7	49.7	135.6
40	83.0	187.0	-29.5	-43.5	54.5	144.2
35	87.2	198.2	-27.3	-45.9	60.9	151.9
30	90.4	209.4	-24.3	-48.0	66.4	159.9

Note: (1) Runs made with spacecraft librations

(2) Data for combined errors includes effect of gyro errors and noise

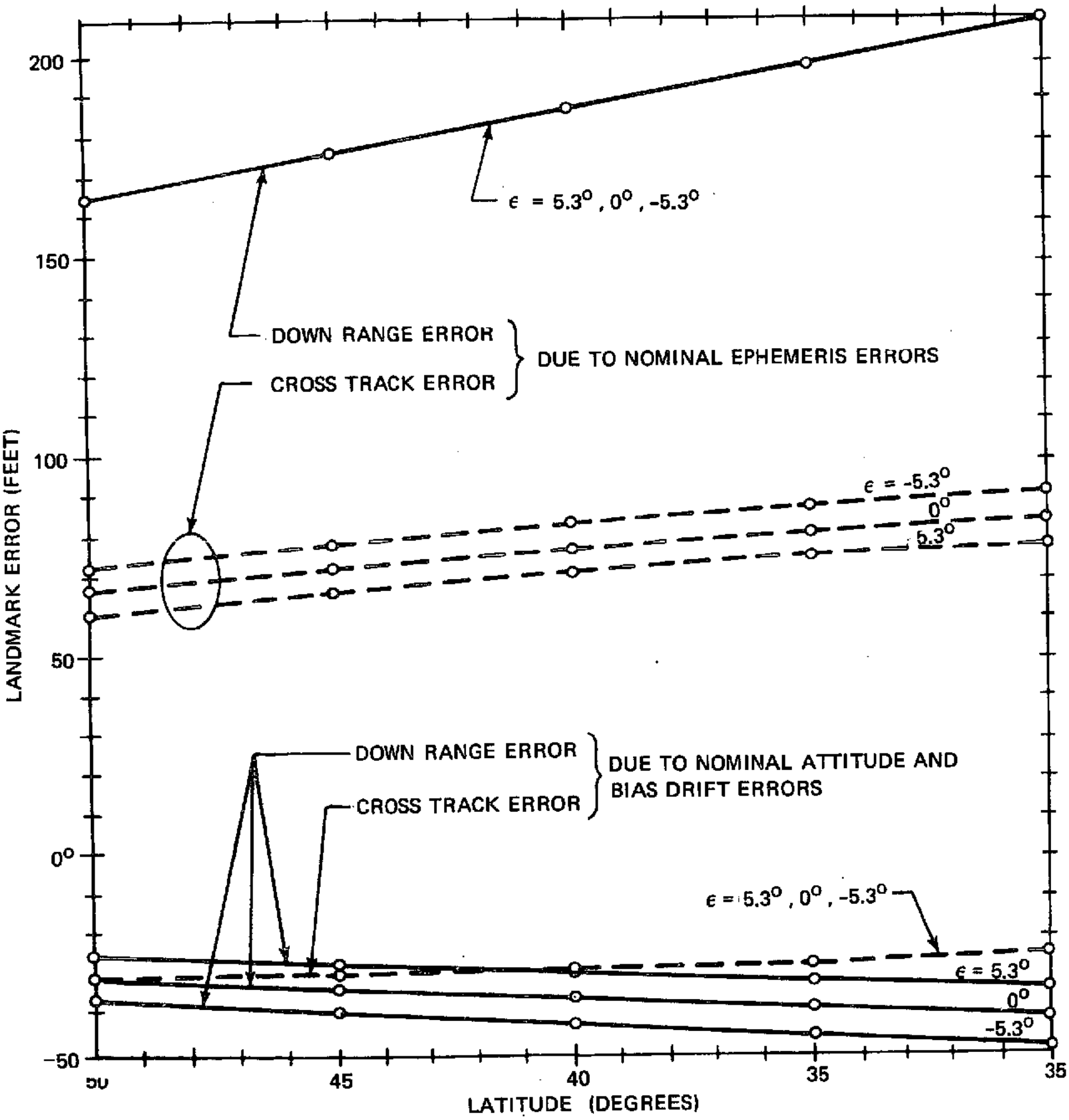


Figure 3-1 Uncorrected Landmark Errors Versus Latitude Due to Nominal Initial Errors in Ephemeris, and in Attitude and Bias Drift.

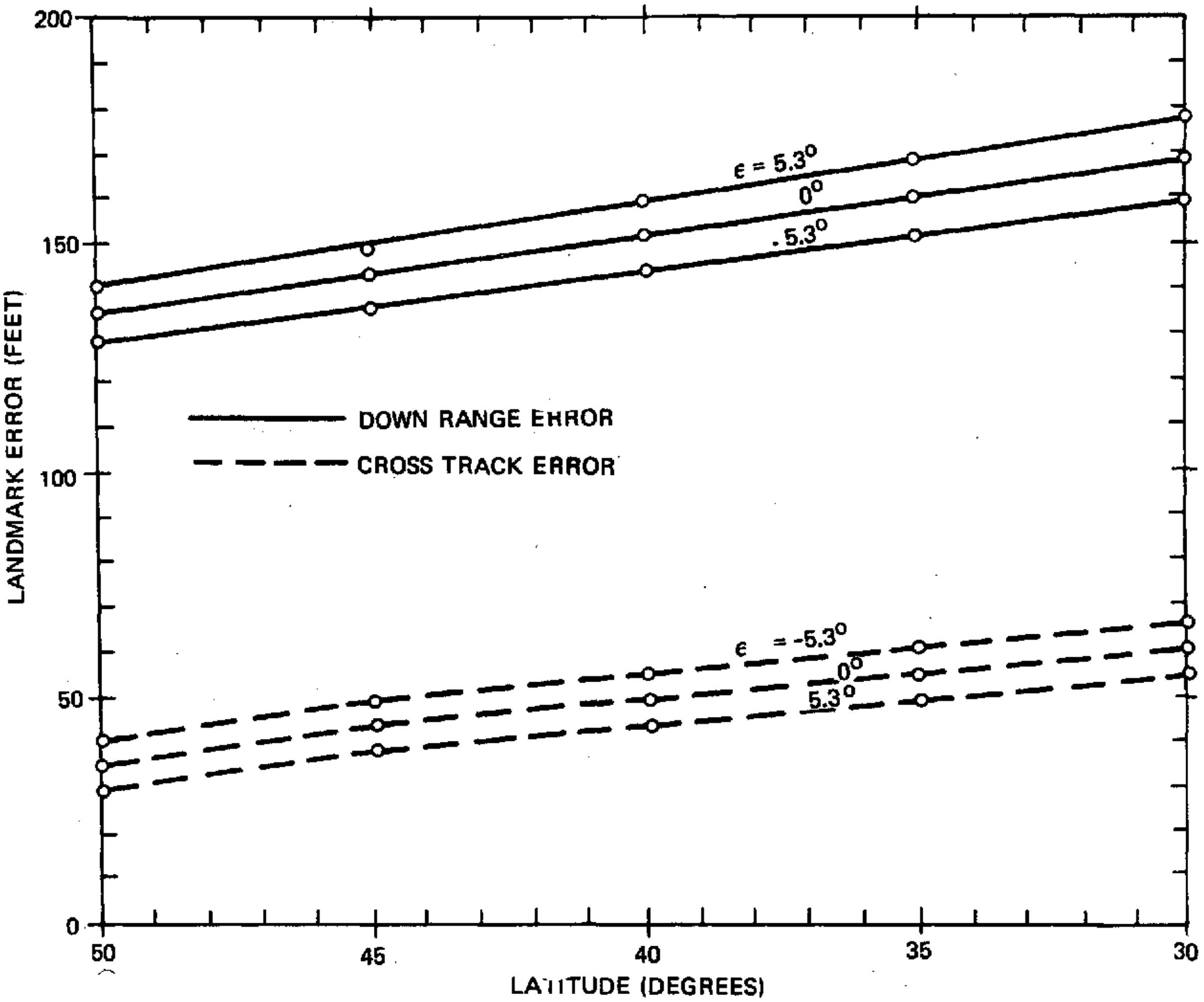


Figure 3-2 Uncorrected Landmark Errors Versus Latitude Due to Nominal Initial Errors in Ephemeris, Attitude, and Bias Drift.

to the ephemeris errors. This result could be of significant value in future studies involving the use of known landmarks to estimate both the spacecraft attitude and orbital ephemeris.

In Table 3-25 the corrected landmark errors are presented for the combined ephemeris and attitude error cases of Table 3-24. For the first time in this report, data is presented showing the effect of using a control point (or known landmark) observed at a latitude other than 50°. In Table 3-25 the corrected landmark error is given for an unknown landmark using the correction data obtained for a control point either at 50°, 40°, or 30° latitude. It is clear from Table 3-25 that the use of a control point at 40° latitude (midpoint of pass) results in a maximum landmark error which is about one half of that obtained when the control point is at 50° or 30° latitude. Graphical illustration of the results for a control point at 40° latitude is shown in Figure 3-3. It should be noted in Figure 3-3 that the polarity of the cross-track error curves was reversed since they would have coincided closely with the down-range error curves.

3.5.2 COMBINED EFFECT OF NOMINAL EPHEMERIS ERRORS AND 3X NOMINAL ATTITUDE ERRORS

As a matter of interest, data was also generated showing the effect of combining nominal ephemeris errors with attitude and gyro bias drift errors which were three times nominal. The data for this case is shown in Table 3-26 and Figures 3-4 and 3-5. Of particular interest is the increase in the spread of the down-range (or range) error curves for different scan beam angles (see Figures 3-2 and 3-5). From the preceding discussion we know that the spread in the down-range error curves is due to the attitude and bias drift errors. In particular, it is due to the presence of a yaw attitude error at the time of observation of the unknown landmark. The yaw error did not affect the control point since $\epsilon = 0$.

Table 3-25

LANDMARK ERRORS VS. LATITUDE FOR NOMINAL INITIAL ERRORS IN EPHEMERIS, ATTITUDE AND BIAS DRIFT

Indicated Latitude (deg)	Landmark Error (feet)							
	At Indicated Latitude		After Correction Using Known Landmark Along $\epsilon = 0^\circ$ at					
			50° Lat.		40° Lat.		30° Lat.	
	Track	Range	Track	Range	Track	Range	Track	Range
$\epsilon = 0^\circ$								
50	34.9	135.1	0	0	-13.8	-16.5	-25.8	-33.3
45	43.9	142.4	9.0	7.3	-4.8	-9.2	-16.8	-26.0
40	48.7	151.5	13.8	16.5	0	0	-12.0	-16.9
35	55.1	159.9	20.2	24.9	6.4	8.4	-5.6	-8.5
30	60.7	168.4	25.8	33.3	12.0	16.9	0	0
$\epsilon = +5.3^\circ$								
50	28.7	141.1	-6.2	6.0	-20.0	-10.5	-32.0	-27.3
45	37.7	149.1	2.9	14.1	-10.9	-2.4	-23.0	-19.3
40	42.6	158.9	7.7	23.8	-6.1	7.3	-18.1	-9.5
35	49.1	168.0	14.2	32.9	0.4	16.5	-11.6	-0.4
30	54.8	176.9	19.9	41.8	6.1	25.4	-5.9	8.5
$\epsilon = -5.3^\circ$								
50	40.7	129.1	5.8	-6.0	-8.0	-22.5	-20.0	-39.3
45	49.7	135.6	14.8	0.5	1.0	-15.9	-11.0	-32.8
40	54.5	144.2	19.6	9.1	5.8	-7.3	-6.2	-24.2
35	60.9	151.9	26.0	16.8	12.2	0.3	0.2	-16.5
30	66.4	159.9	31.5	24.8	17.7	8.4	5.7	-8.5

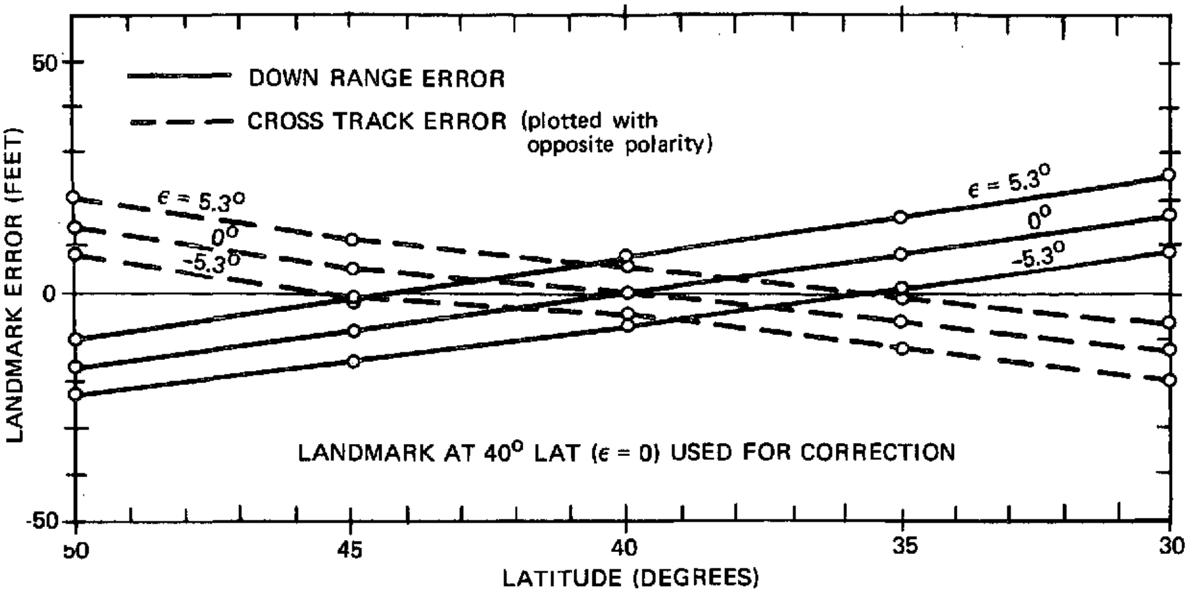


Figure 3-3 Corrected Landmark Errors Versus Latitude for Nominal Initial Errors in Ephemeris, Attitude and Bias Drift.

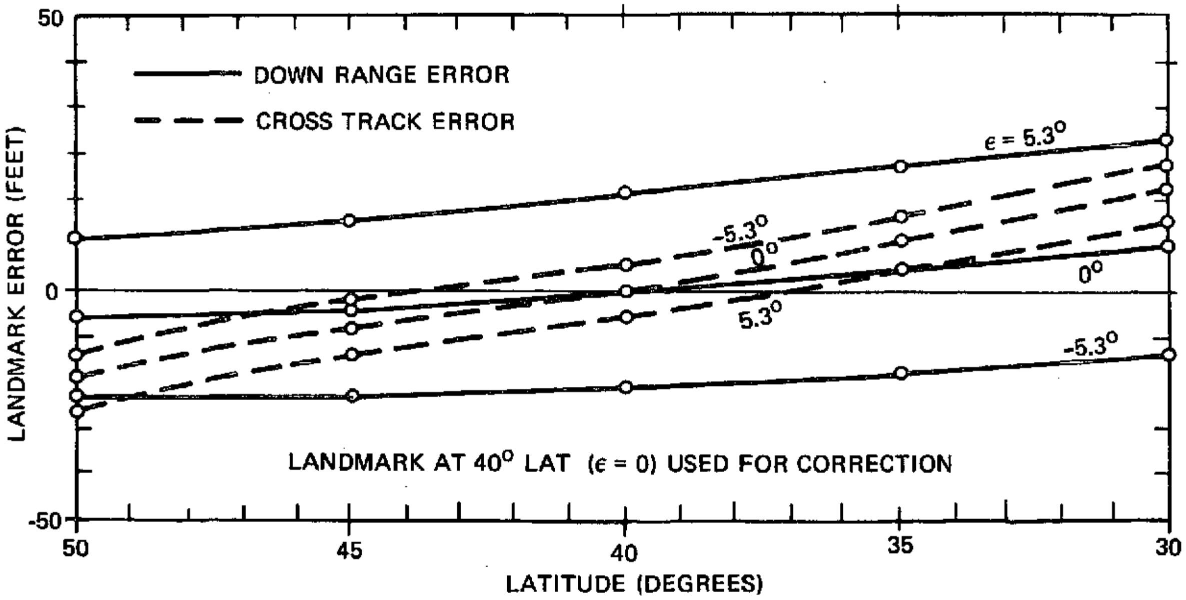


Figure 3-4 Corrected Landmark Errors Versus Latitude for Nominal Initial Ephemeris Errors and 3X Nominal Attitude and Bias Drift Errors.

Table 3-26

LANDMARK ERRORS VS. LATITUDE FOR NOMINAL INITIAL EPHEMERIS ERRORS AND 3X NOMINAL ATTITUDE AND BIAS DRIFT ERRORS

Indicated Latitude (deg)	Landmark Error - feet							
	At Indicated Latitude		After Correction Using Known Landmark along $\epsilon = 0^{o}$ at					
			50^{o} Lat.		40^{o} Lat.		30^{o} Lat.	
	Track	Range	Track	Range	Track	Range	Track	Range
$\epsilon = 0$								
50	-28.8	71.4	0	0	-19.2	- 6.6	-41.0	-16.2
45	-17.3	73.6	11.4	2.2	- 7.8	- 4.5	-29.6	-14.1
40	- 9.6	78.1	19.2	6.6	0	0	-21.8	- 9.6
35	1.0	82.5	29.7	11.1	10.5	4.4	-11.3	- 5.2
30	12.3	87.7	41.0	16.2	21.8	9.6	0	0
$\epsilon = +5.3^{o}$								
50	-35.4	89.2	-6.6	17.8	-25.8	11.2	-47.6	1.6
45	-23.8	93.1	5.0	21.6	-14.2	15.0	-36.0	5.4
40	-16.0	98.9	12.8	27.5	- 6.4	20.9	-28.2	11.2
35	- 5.5	104.9	23.2	33.5	4.1	26.9	-17.8	17.2
30	5.8	111.4	34.5	39.9	15.3	33.3	- 6.5	23.7
$\epsilon = -5.3^{o}$								
50	-23.6	53.6	5.1	-17.8	-14.1	-24.4	-35.9	-34.0
45	-12.3	54.1	16.5	-17.3	- 2.7	-23.9	-24.6	-33.5
40	- 4.4	57.2	24.3	-14.3	5.1	-20.9	-16.7	-30.5
35	6.3	60.0	35.0	-11.4	15.8	-18.0	- 6.0	-27.6
30	17.7	64.0	46.5	- 7.5	27.3	-14.1	5.5	-23.7

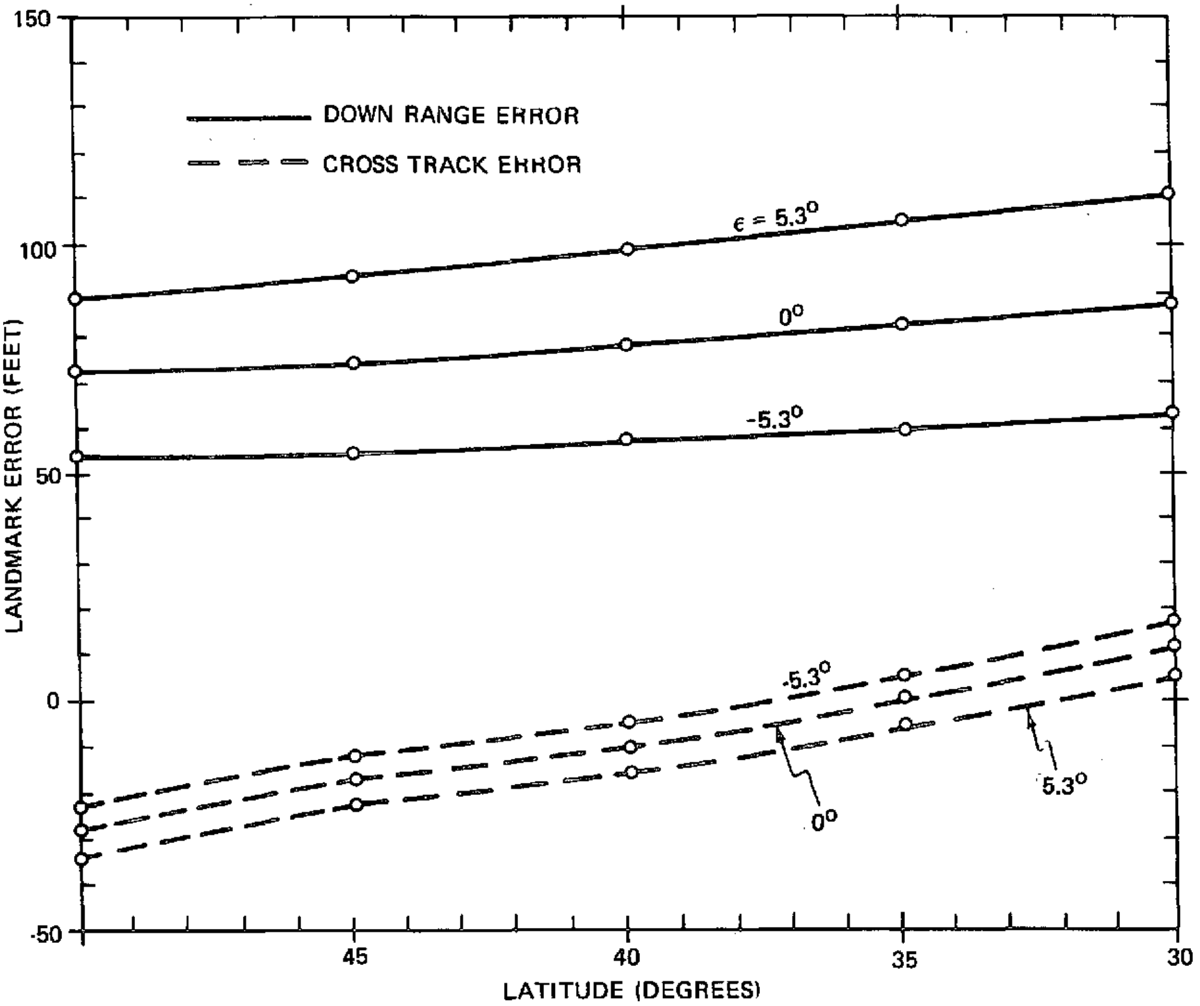


Figure 3-5 Uncorrected Landmark Errors Versus Latitude for Nominal Initial Ephemeris Errors and 3X Nominal Attitude and Bias Drift Errors.

It is important to note that Figures 3-3 and 3-4 can be used to obtain similar data for the case where the control point is located at a latitude other than 40° but still observed at $\epsilon = 0$. For example, to obtain the results for the case where the control point is at 50° latitude, one simply translates vertically the curves in the figure until the down-range and cross-track error curves for $\epsilon = 0$ pass through zero for 50° latitude.

Table 3-27 gives data on uncorrected and corrected landmark errors for various error configurations. The important things to note here are: (1) The landmark errors after correction are three to five times greater with nominal ephemeris errors than with nominal attitude errors, when these are considered separately. (2) The relative effect on landmark errors after correction is much less when the nominal attitude errors are increased by a factor of three then when the nominal ephemeris errors are increased by the same factor.

Table 3-27

UNCORRECTED AND CORRECTED LANDMARK ERRORS FOR VARIOUS ERROR CONFIGURATIONS

Nominal Errors for				Landmark Error (feet)				Landmark Error After Correction (ft)	
				At Start (Lat= 50^{o})		At End (Lat= 30^{o})			
Ephem- eris	Attitude & Bias Drift	Gyro SFE & IA Mlm.	Gyro Drift & Quant. Noise	Track	Range	Track	Range	Track	Range
1X	0	0	0	65.6	164.6	84.6	209.4	19.0	44.8
1X	0	1X	1X	66.7	166.9	84.9	208.8	18.2	41.9
0	1X	0	0	-31.8	-31.8	-24.2	-40.4	7.6	-8.5
0	1X	1X	1X	-30.7	-29.5	-23.8	-41.0	6.9	-11.4
0	0	1X	1X	1.1	2.3	0.4	-0.6	-0.7	-2.9
1X	1X	1X	1X	34.9	135.1	60.7	168.4	25.8	33.3
1X	3X	1X	1X	-28.8	71.4	12.3	87.7	41.0	16.2
3X	1X	1X	1X	166.1	464.3	229.8	587.1	63.7	122.9
3X	3X	1X	1X	102.4	400.6	181.4	506.4	78.9	105.8
Following runs are without librations									
1X	1X	1X	1X	34.9	134.5	67.0	170.7	32.1	36.3
1X	0	0	0	65.6	164.0	84.4	210.0	18.8	46.0
0	1X	0	0	-31.8	-31.8	-22.8	-40.4	9.0	-8.5

Note (1): 3X denotes that nominal errors are multiplied by a 3 times factor, etc.

(2): Correction is made assuming known landmark (or control point) is located at 50^{o} latitude and along $\epsilon = 0$.

(3): Runs with spacecraft librations unless otherwise noted.

(4): $\epsilon = 0$.

(5): Runs with gyro noise made for a particular random run.

SECTION 4

CONCLUSIONS

The primary objective of the landmark location study was to determine the effects of errors in spacecraft orbital ephemeris, spacecraft attitude, and gyro bias drift on the ability to determine the locations of unknown landmarks with respect to known landmarks (or control points) in the payload sensor imagery. Most of the data was generated for observational passes over the continental USA. It is felt that sufficient data was generated to clearly show the sensitivity of landmark location determination to errors in each component of the orbital ephemeris, spacecraft attitude, and gyro bias drift.

In the present study it was assumed that only one known landmark (or control point) was observed during a pass over the USA. The data in this report shows what the effects of ephemeris and attitude errors are on determining the locations of other arbitrary features (observed during the pass) with respect to this control point. In most cases the control point was arbitrarily selected to be on the northern USA border in order to show the maximum error in determining the relative location of an unknown feature (or landmark) on the southern border. It is important to note that errors of about the same magnitude would occur if the control point was on the southern border and the unknown landmark was on the northern border. As shown in Section 3.5.2, the error for an unknown landmark on the northern or southern border would have been about half as large if the control point had been located halfway across the USA (i.e. at 40° latitude). It is obvious that the relative landmark errors can be reduced by using more than one control point in an observational pass. The data in Section 3 provides some indication of how close the control point must be to the unknown feature in order to achieve a certain minimum accuracy in relative mapping. However, it should be pointed out that some of the data in Section 3 does not give a statistical indication of performance since the results are for discrete sets of errors in orbital ephemeris, satellite attitude, and gyro bias drift. A complete statistical analysis would probably be appropriate in the future after better statistical data has been obtained for the orbital ephemeris errors (including correlation between the errors), and possibly after a particular SIMS configuration has been identified for this type of mission.

One surprising result of this study is the apparent decrease in the effect of certain attitude and bias drift errors on relative landmark location determination when these errors happen to be the errors in the smoothed estimates of attitude and bias drift. By using smoothed (or filtered) estimates of attitude and gyro bias drift, one takes advantage of certain relationships established between the errors in these estimates which tend to decrease the effect of certain errors on relative landmark location determination. This was clearly shown to be the case for errors in yaw angle and roll bias drift for SIMS-A in Section 3.4. There it was shown that these two errors tend to cancel each other out. At this point it would seem appropriate to review the history associated with the yaw attitude error in the overall SIMS trade study. At the beginning of this study the desired accuracy in estimating pitch, roll, and yaw was 0.001° (1σ) each, although it was felt by many that the yaw accuracy did not have to be as good as that for pitch and roll since yaw error had very little effect on landmarks observed near local vertical. In the SIMS-A studies[169, 177] it was found that the desired yaw accuracy was also the most difficult to achieve because of the nature of that system. In Section 3.2 it is shown that the yaw error indeed does not introduce as much landmark error as pitch and roll when the position of that landmark is being determined solely with satellite position and attitude data. However, when one attempts to determine the location of an unknown landmark with respect to a known landmark, the landmark error is more sensitive to yaw error than it is to pitch or roll error. Note that these are the results obtained for the individual errors in pitch, roll, and yaw. In looking at the individual sensitivity data of Section 3.2, it would therefore seem that the relatively larger yaw errors of SIMS-A would make a significant contribution to the relative landmark error. Yet, this is shown in Section 3.4 to not be the case when the attitude and bias drift errors are those obtained as the result of smoothing. This conclusion would seem to be a very good argument for having a SIMS onboard the satellite for this type of mission, since this would permit smoothed estimates of attitude and bias drift to be obtained. Although no smoothed data was available for ephemeris errors, it would also be interesting to see what effect such data would have on landmark errors. It is possible that smoothed estimates of satellite position and velocity may have an effect similar to that obtained for smoothed estimates of attitude and gyro bias drift.

Another important result of this study is that obtained on the effect of higher harmonics in the Earth's gravitational field. For landmark location determination in a single pass over the continental USA, it is shown in Section 3.3 that a maximum landmark error of about 10 feet will occur when using a gravitational model consisting of the central force field and only the well known harmonic J_2. The above error is that for an unknown landmark on the southern or northern border when the known landmark (or control point) is on the opposite border. If the control point happens to be mid-way across the USA (40° latitude), the above error will be half as large. Sufficient data is presented in Section 3.3 to show the separate effects of additional harmonics when greater accuracy is desired. The results in Section 3.3 show that the effect of the uncertainties in the harmonic coefficients is much smaller than that of the harmonic coefficients themselves.

SECTION 5

RECOMMENDED FUTURE STUDIES

The present landmark location study was primarily concerned with determining the effects of spacecraft orbital ephemeris and attitude errors on the ability to determine the locations of unknown landmarks with respect to known landmarks when a relatively simple but effective technique of landmark correction is used. Although the results for each case in this report are for just one known landmark per observational pass, it is felt that sufficient data was generated to show the effects of known landmark location at different points in the imagery. No attempt was made to use an optimal technique of correction for relative landmark location since this was beyond the scope of this study and was considered to be more appropriate in future studies.

Although sufficient sensitivity data was generated in the present study to enable one to estimate what the performance would be for various combinations of errors in orbital ephemeris and satellite attitude, it is possible that additional data may be desired in the future using errors which are more realistic than those defined to be nominal in the present effort. This would, of course, depend on decisions as to what SIMS configuration is adopted. Also, as indicated in Section 4, it would be interesting to see what the effect would be on landmark location determination when smoothed estimates of orbital ephemeris are used.

One area which is strongly suggested in future studies is the use of known landmarks in the imagery to update the orbital position and velocity, the satellite attitude, and the gyro bias drift. This represents an update of twelve parameters and should be done with the Kalman filter and/or an optimal smoother. It is felt that this approach may indirectly provide the optimal technique of landmark correction mentioned earlier.

Such a study should initially be a covariance analysis with consideration being given to various gyro package configurations, landmark distributions, etc. Consideration should also be given to the addition of star measurements by various candidate star sensors. Eventually, a full state simulation should be performed on the most promising configuration (or configurations) with optimal estimates being made of both the state and the covariance matrix of the state estimation error.

In addition to the above, the Active and Passive Landmark Mechanization Study, which was not performed in the present study effort, should be done in the near future because of its possible value to automatic ground data processing of thematic mapper imagery.

APPENDIX A

ERRATA FOR FINAL REPORT R-741

The following corrections should be made in the Final Report[177] of which this report is an addendum:

Page	Correction
2-4	The value of ω_L given at top of page should be 0.478 instead of 0.005. Also remove ' -3σ '.
	In Equation 2-3 draw line under ω_T to denote vector.
2-16	Place dot above P on left side of Equation 2-32.
2-17	Place dot above U on left side of Equation 2-36.

SECTION 6

REFERENCES

1. through 84. (These are the references contained in reference 85.)

85. (Included here for convenience.) G. Ogletree with J. Coccoli, R. McKern, M. Smith, R. White, "Interim Technical Report No. 1, Candidate Configuration Trade Study, Stellar-Inertial Measurement System (SIMS) for an Earth Observation Satellite (EOS), " Report E-2616, MIT/CSDL, 5 November 1971.

85. through 140. (These are the references contained in reference 141.)

141. G. Ogletree, J. Coccoli, R. McKern, M. Smith and R. White, "Interim Technical Report No. 2, Candidate Configuration Trade Study, Stellar-Inertial Measurement System (SIMS) for an Earth Observation Satellite (EOS)," Report E-2630, MIT/CSDL, 31 January 1972.

141. through 168. (These are the references contained in reference 169.)

169. G. Ogletree, J. Coccoli, R. McKern, M. Smith and R. White, "Interim Technical Report No. 3, Candidate Configuration Trade Study, Stellar-Inertial Measurement System (SIMS) for an Earth Observation Satellite (EOS)," Report E-2651, MIT/CSDL, 15 June 1972.

SECTION 6

REFERENCES (Contd)

170. through 176. (These are the references contained in reference 177.)

177. R. White, "Final Report, Candidate Configuration Trade Study Stellar-Inertial Measurement System (SIMS) for an Earth Observation Satellite (EOS)," Report R-741, MIT/CSDL, 31 January 1973.

178. S. Pines, G. Austin, "Gravitational Acceleration of a Point Mass due to a Rotating Nonspherical Body," Report No. 69-12, Analytical Mechanics Associates, Westbury, N. Y., May 1969.

179. C. A. Lundquist, "Geodetic Satellite Results during 1967," Special Report 264, Smithsonian Astrophysical Observatory, 29 December 1967.

180. E. M. Gaposchkin, K. Lambeck, "1969 Smithsonian Standard Earth (II)," Special Report 315, Smithsonian Astrophysical Observatory, 18 May 1970.

181. Y. Kozai, "Revised Zonal Harmonics in the Geopotential," Special Report 295, Smithsonian Astrophysical Observatory, February 1969.

182. Y. Kozai, "Seasonal Variations of the Geopotential," Special Report 312, Smithsonian Astrophysical Observatory, April 1970.

183. P. Mitchell, "Preliminary MSFN Error Analysis for AS-504A," MSC Int. Note No. 67-FM-29, Mission Planning and Analysis Div., Manned Spacecraft Center, Houston, Texas, 1 March 1967.

SECTION 6

REFERENCES (Contd)

184. P. Mitchell, "C-Mission Navigation Error Analysis," MSC Int. Note No. 68-FM-114, Mission Planning and Analysis Div., Manned Spacecraft Center, Houston, Texas, 18 April 1968.